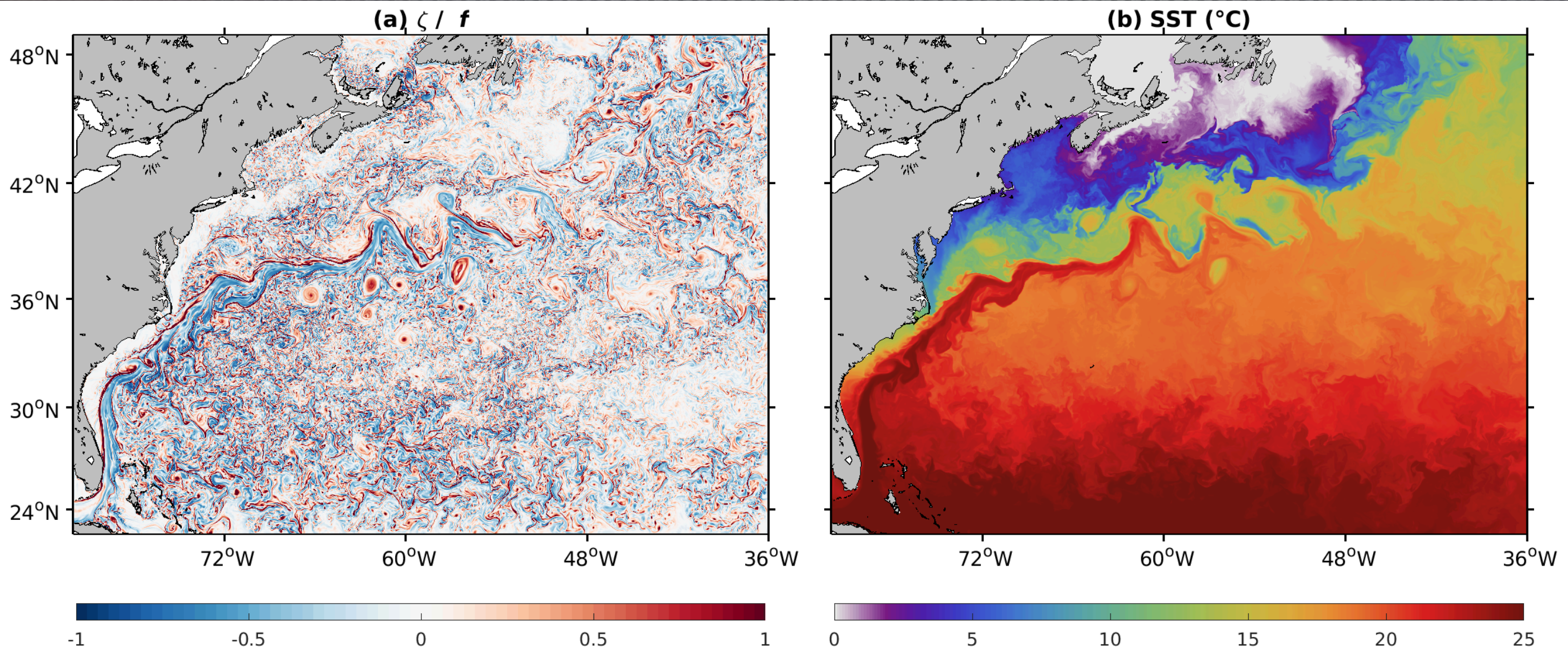


Improved Gulf Stream separation through Brinkman penalization

P. Marchesiello, L. Debreu, N. Kevlahan

(Debreu et al., Ocean Modelling 2020, 2022)

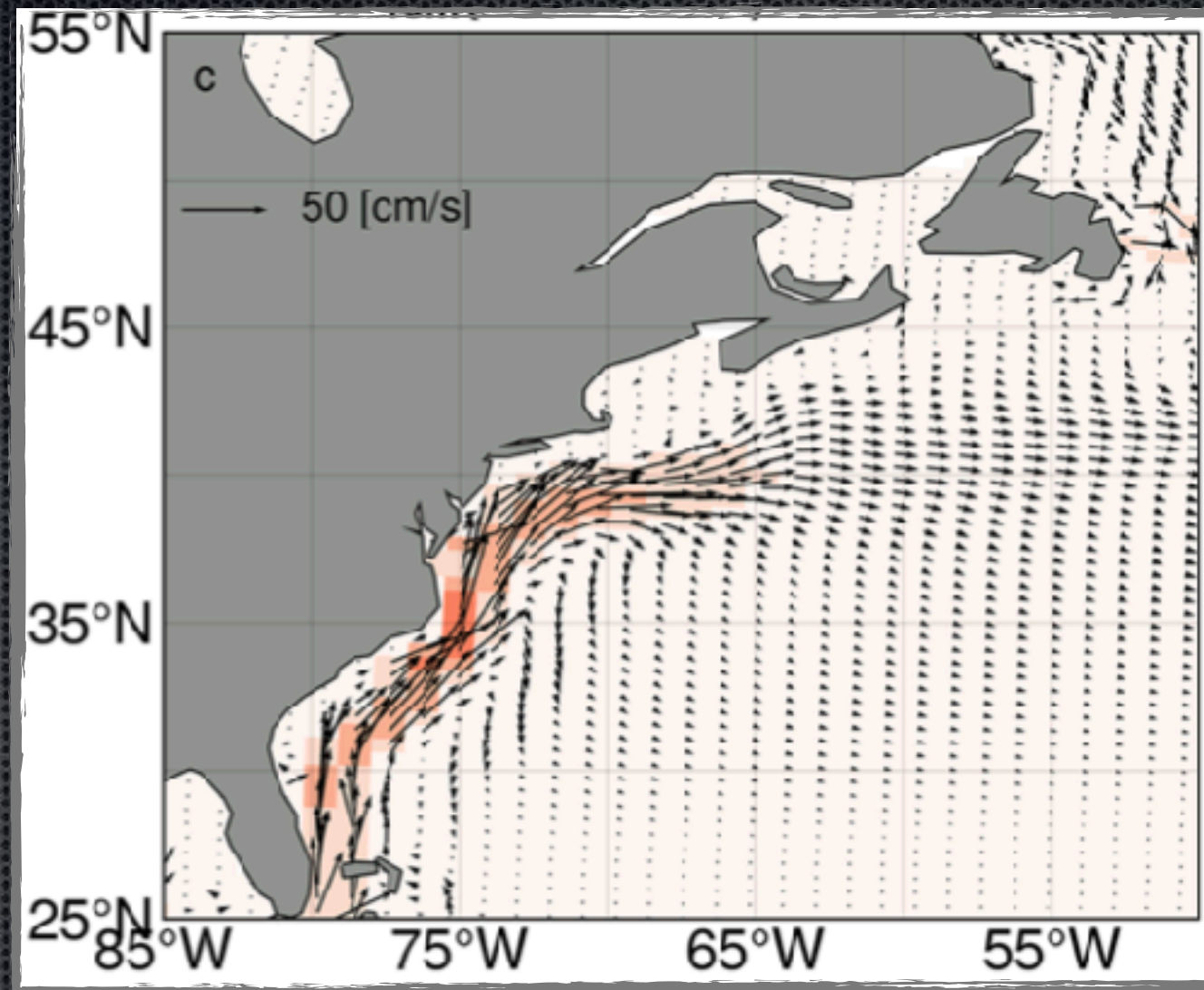
Realistic GS simulations at high resolution



Contreras et al. (2022)

2 km resolution
(submesoscale-permitting)

Low-resolution GS simulations



POP (CESM) 1° (Li et al. 2022)

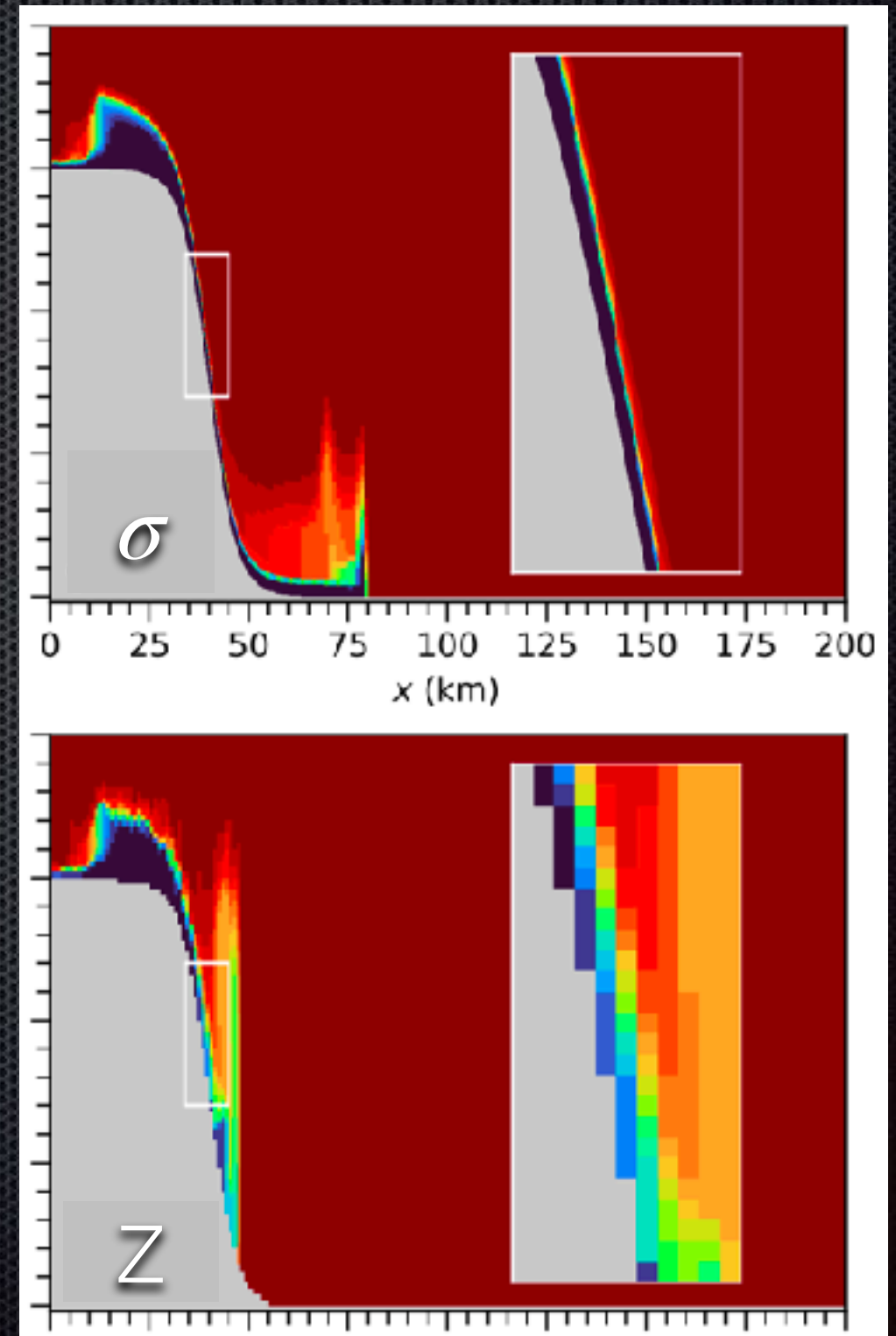
What's wrong with topographic slope at low resolution?

- Topography smoothing is needed in σ models to limit pressure gradient errors:

$$r = \frac{\Delta h}{h} = \frac{S \Delta x}{h} < r_0 \sim 0.2$$

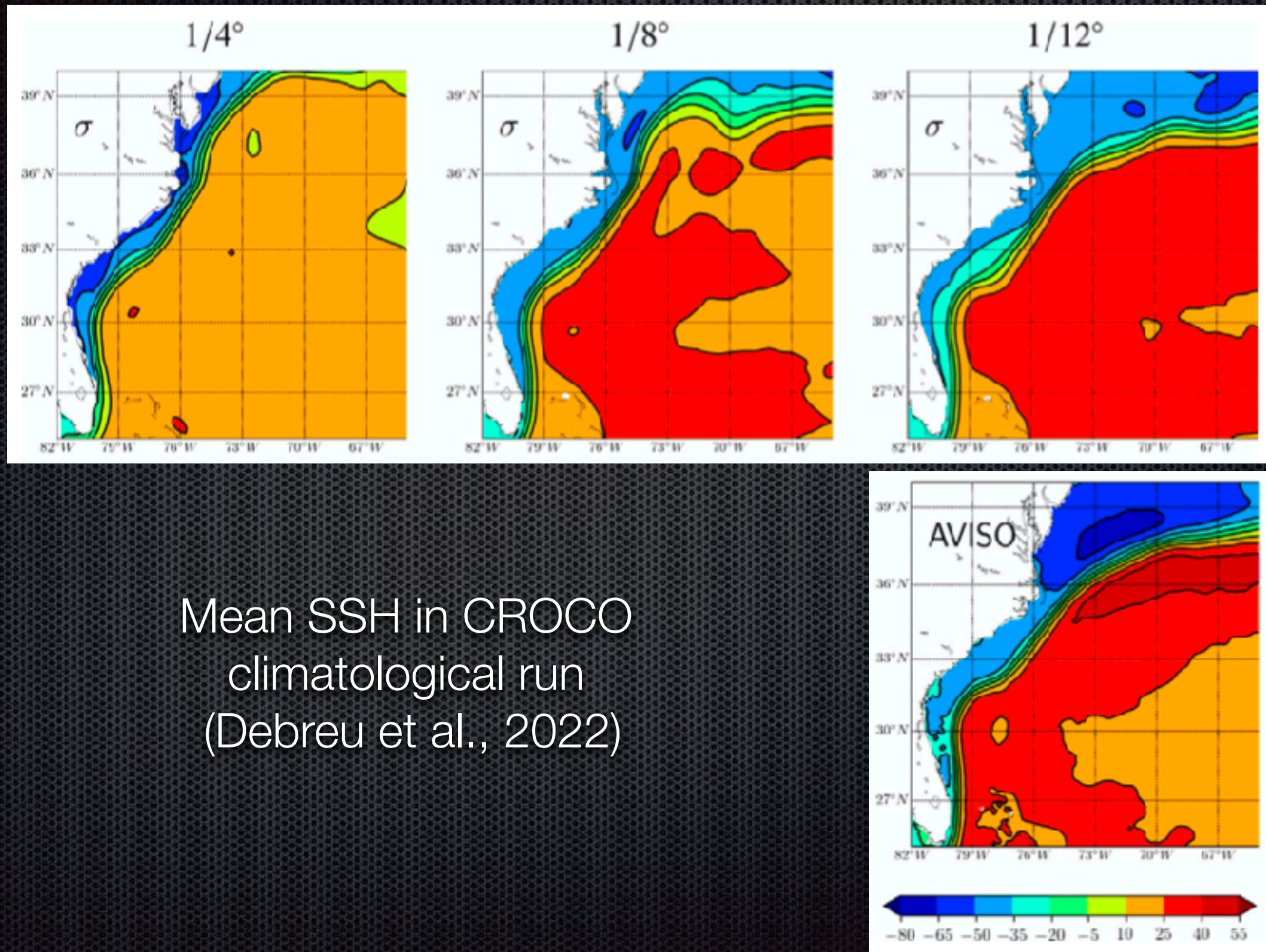
- Deterioration in both σ - and \mathbf{z} -coordinate systems, and z-models have weaker flow-topography interaction (Penduff et al., 2002):

$$J(P_b, h) \sim -fw_b$$



Nasser (2023)

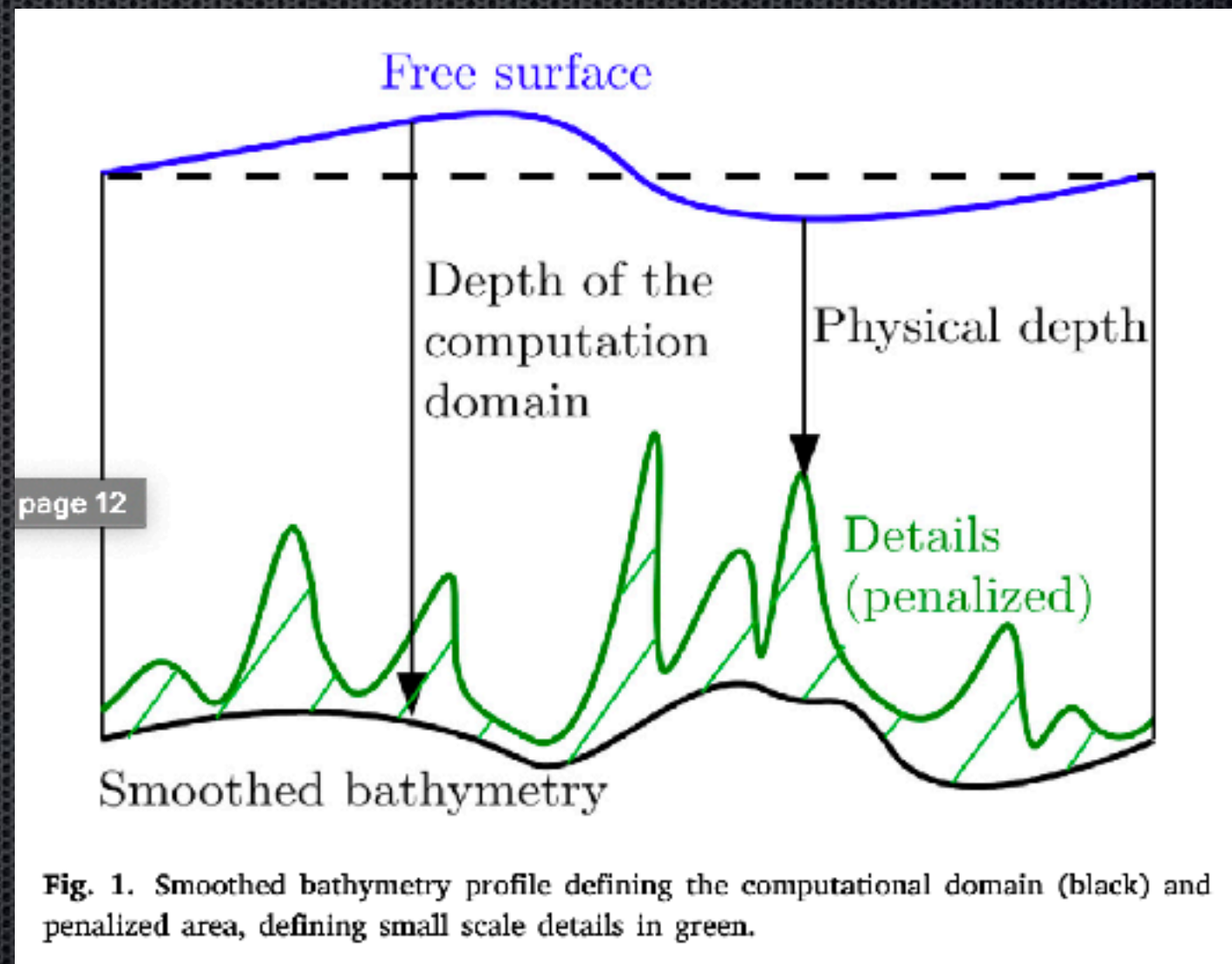
Resolution sensitivity using sigma coordinates



Brinkman Volume Penalization method (immersed boundary conditions)

Debreu et al., OM 2022

Volume penalization is an elegant way to impose Neumann (blocking) and Dirichlet (drag) boundary conditions in discretized PDEs



Penalization method

Debreu et al., OM 2020, 2022

drag due to
permeability ϵ



$$\frac{\partial \tilde{h}u}{\partial t} + \frac{\partial \tilde{h}u^2}{\partial x} + \frac{\partial \tilde{h}u\Omega}{\partial s} = -\tilde{h} \left(g \frac{\partial \eta}{\partial x} + \frac{1}{\rho_0} \frac{\partial p_h}{\partial x} + \frac{g\rho}{\rho_0} \frac{\partial z}{\partial x} \right) - \frac{1}{\epsilon} f(x, z) \tilde{h}u$$

$$\frac{\partial \tilde{h}}{\partial t} + \frac{\partial \tilde{h}u}{\partial x} + \frac{\partial \tilde{h}\Omega}{\partial s} = 0$$

ϵ : permeability



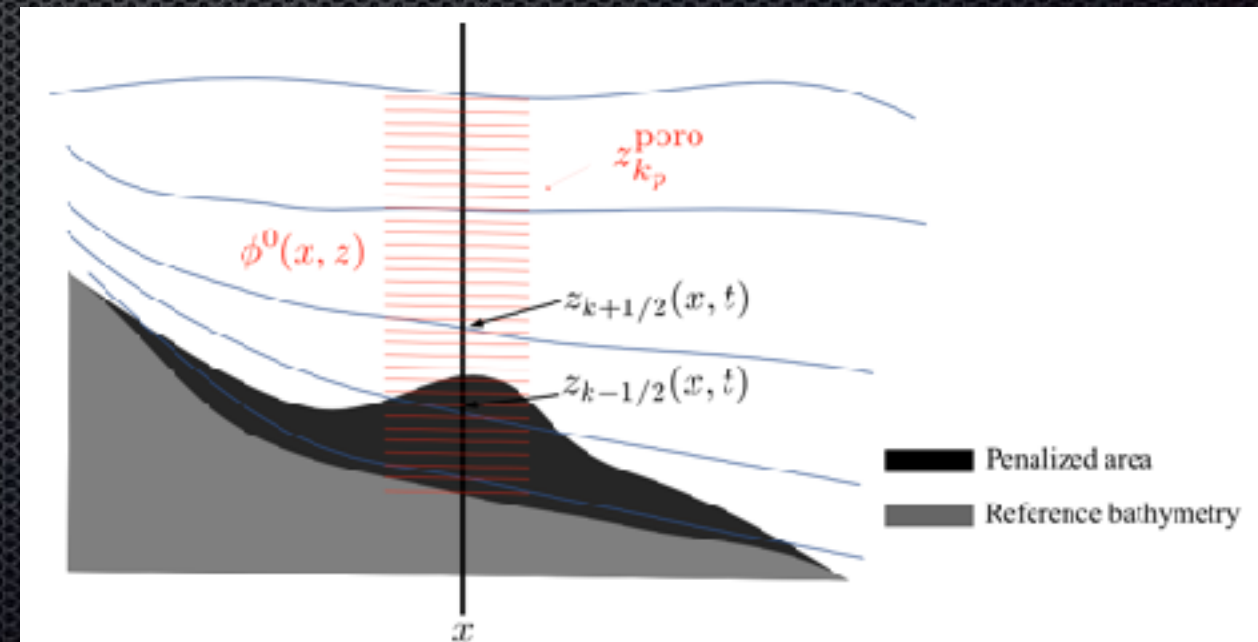
Drag

α : porosity ($\tilde{h} = \alpha h$)



Blocking

$f(x, z)$: mask defining the
penalized region = $f(\tau_b)$

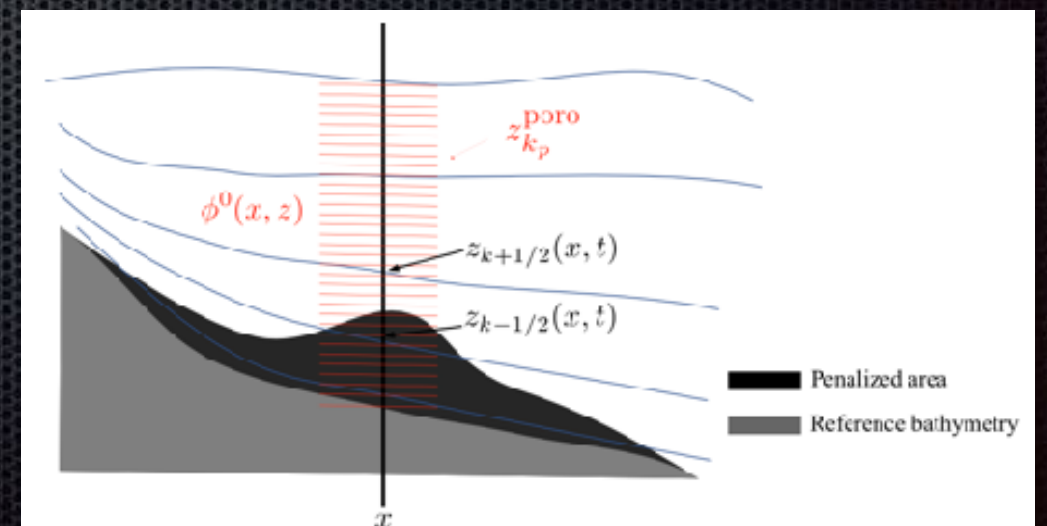


Numerical implementation

The principal challenges were:

- ✓ Conservation of total energy : ventilation of penalized Hz
- ✓ Stability of the drag term (stiff term) : $\epsilon \sim f(\Delta t)$
- ✓ Accounting for penalization in the correction of the barotropic–baroclinic mode splitting (to avoid spurious flow in the solid region)

In practice, the penalized simulations run nearly as fast as the original model simulations



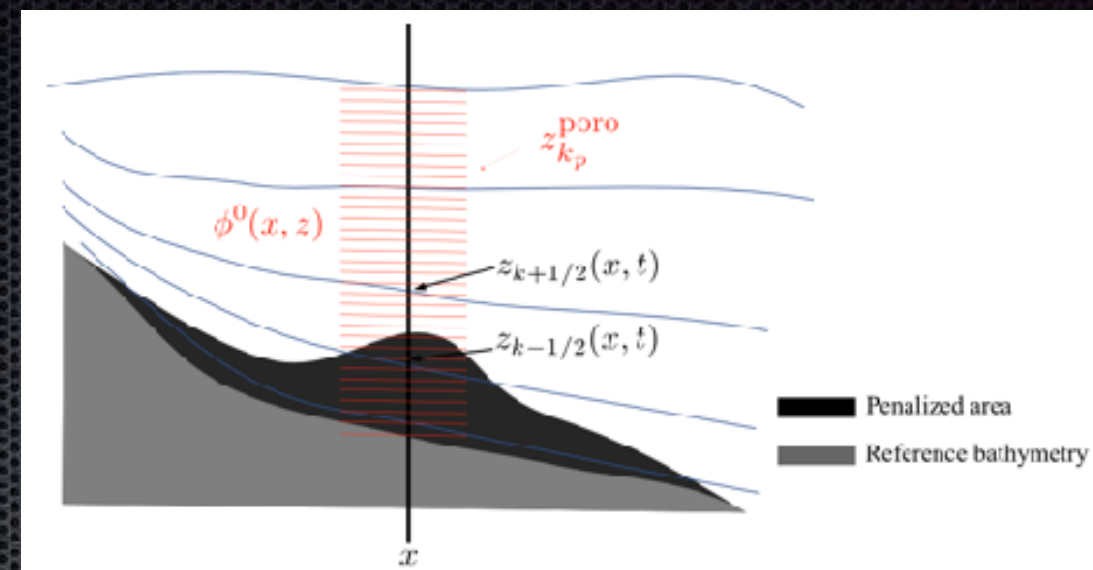
► Code :

► set_depth_porosity :

- Hz, h -> Hzf, hgrid
- hraw -> h_pena_target (get_grid, ana_grid)
- Hz, h = \sum Hz

► set_depth_permeability:

- pena_r
- pena_u
- pena_v



Splitting method

$$\frac{\partial u}{\partial t} = F(u) - \frac{1}{\epsilon} \mathbb{1}(z)u.$$

A splitting method can be used to integrate (12) in two steps,

$$(1) \frac{\partial u}{\partial t} = F(u), \quad (2) \frac{\partial u}{\partial t} = -\frac{1}{\epsilon} \mathbb{1}(z)u.$$

$$u^{n+1} = (1 - \text{pena_u}) u^{n+1,*}$$

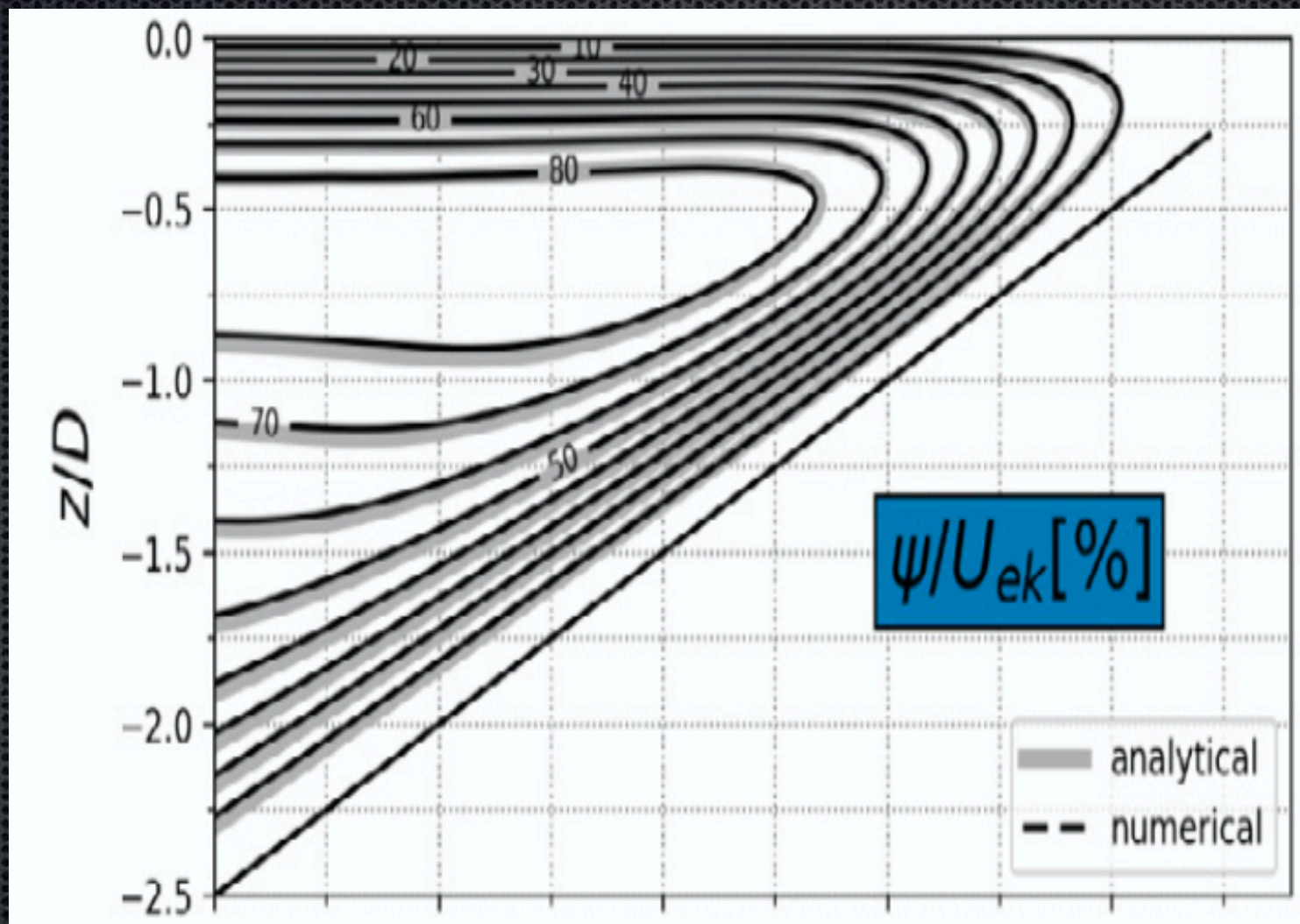
```

do j=Jstr,Jend
# if defined PERMEABILITY
  if (nnew == 3) then
    do k=1,N
      do i=IstrU,Iend
        ru(i,j,k)=ru(i,j,k)*(1.-pena_u(i,j,k))
      enddo
    enddo
  endif
# endif
  do i=IstrU,Iend
    rufrc(i,j)=ru(i,j,1)
  # ifndef BODYFORCE
  # ifdef BSTRESS_FAST
    & + sustr(i,j)*om_u(i,j)*on_u(i,j)
  # else
  # if defined PERMEABILITY
    & + sustr(i,j)*om_u(i,j)*on_u(i,j)
  # else
    & +(sustr(i,j)-bustr(i,j))*om_u(i,j)*on_u(i,j)
  # endif
  # endif
# endif

```

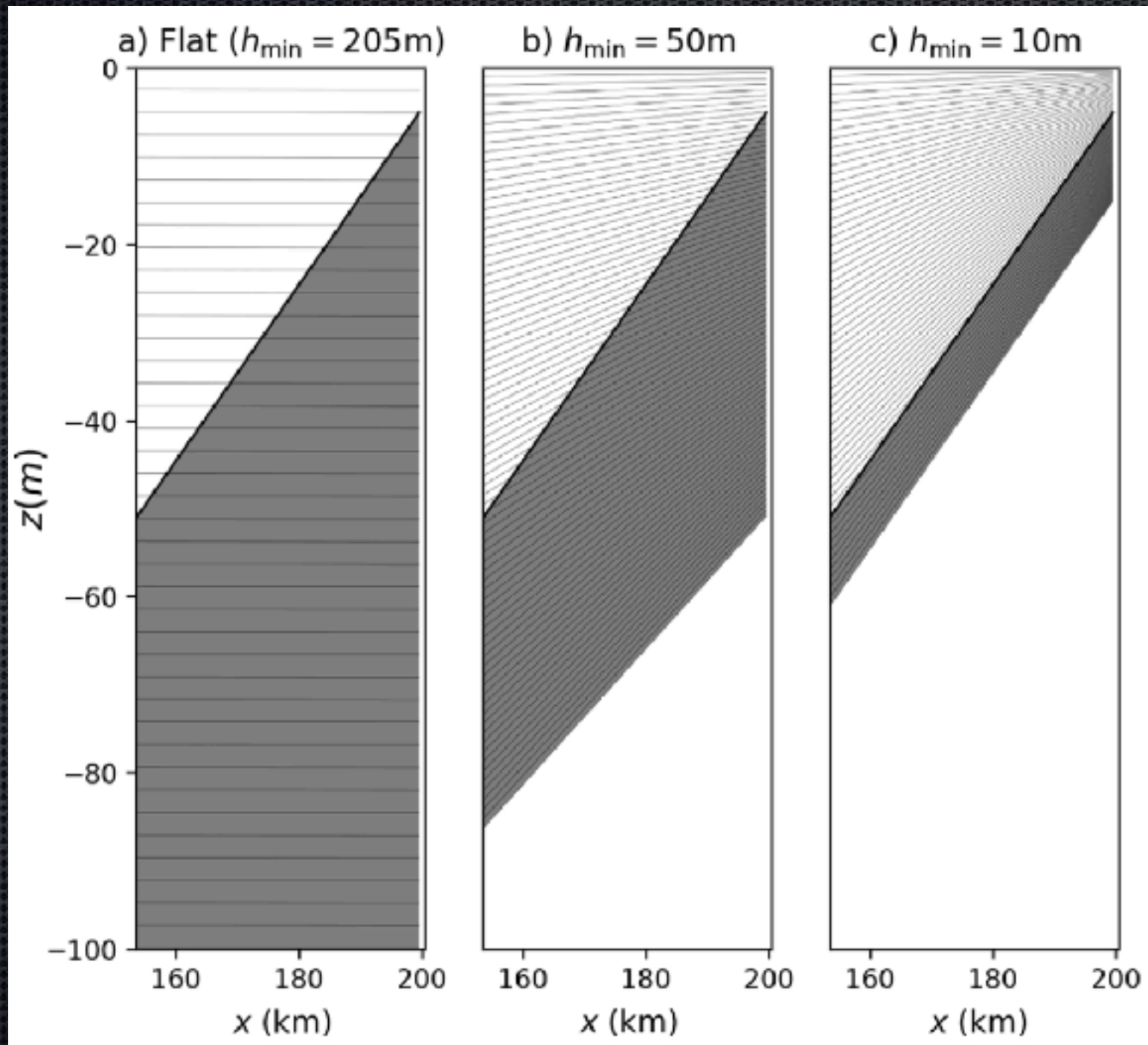

Upwelling test case

Debreu et al. (2020)



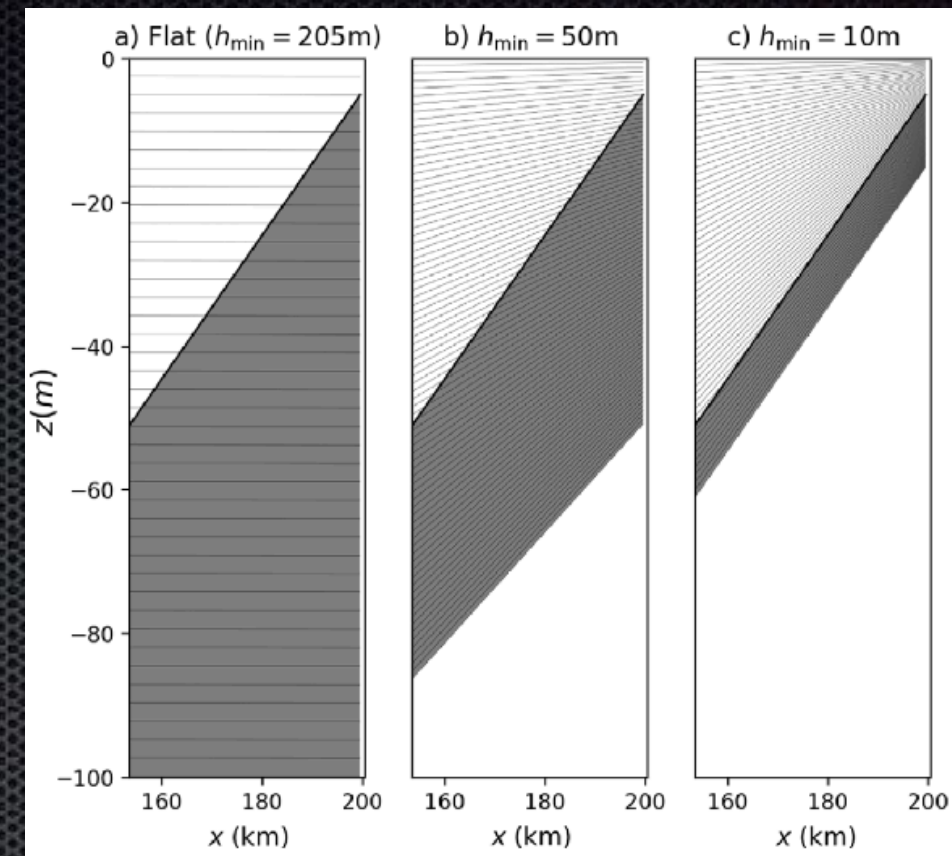
Upwelling test case

Debreu et al. (2020)

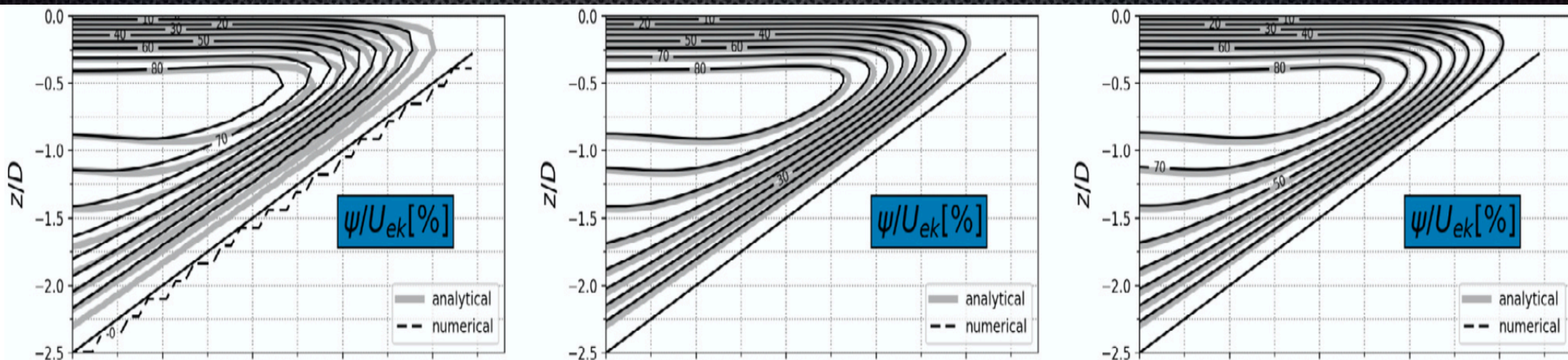


Upwelling test case

Debreu et al. (2020)



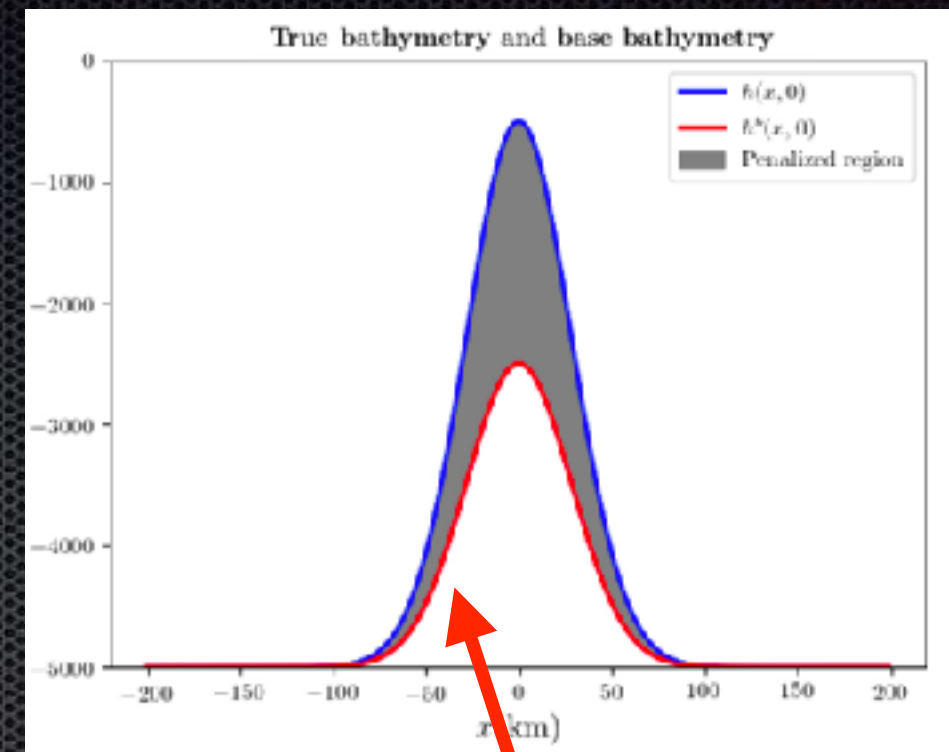
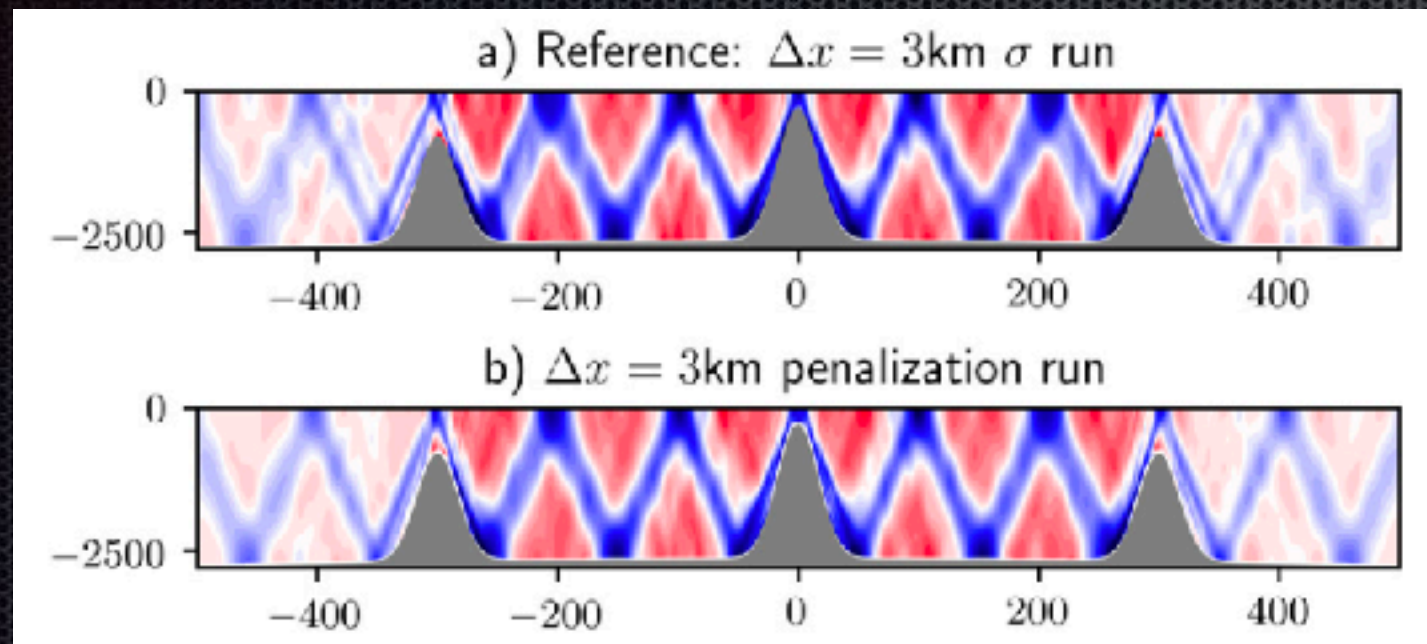
large penalization regions impact resolution but do not show staircase effects



Internal tide test case

Debreu et al. (2020)

u (m/s)

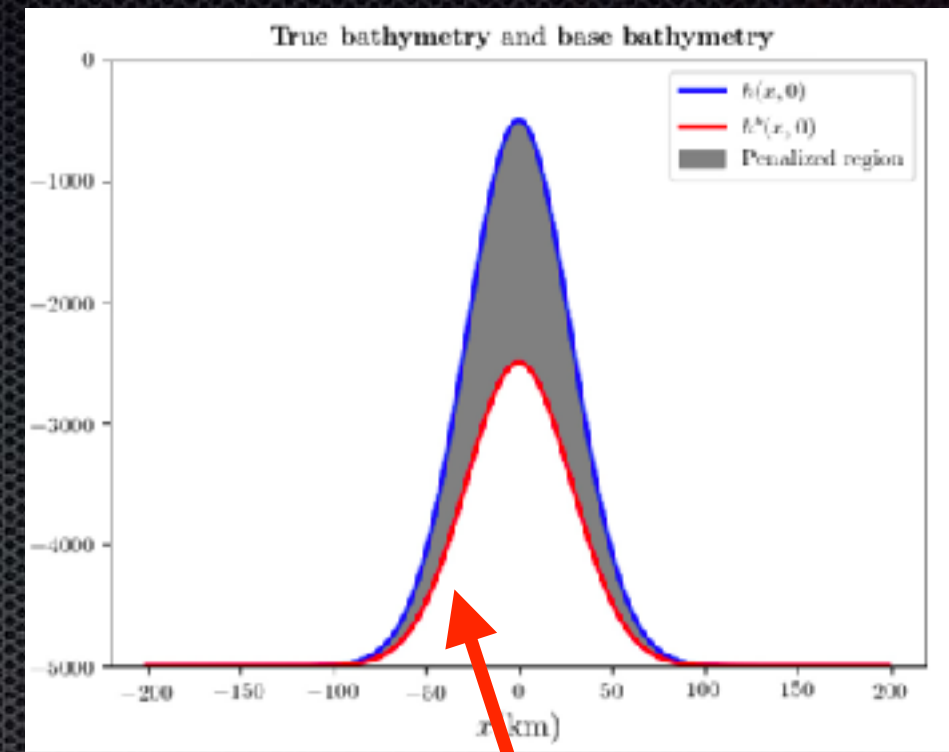


← high-resolution

Internal tide test case

Debreu et al. (2020)

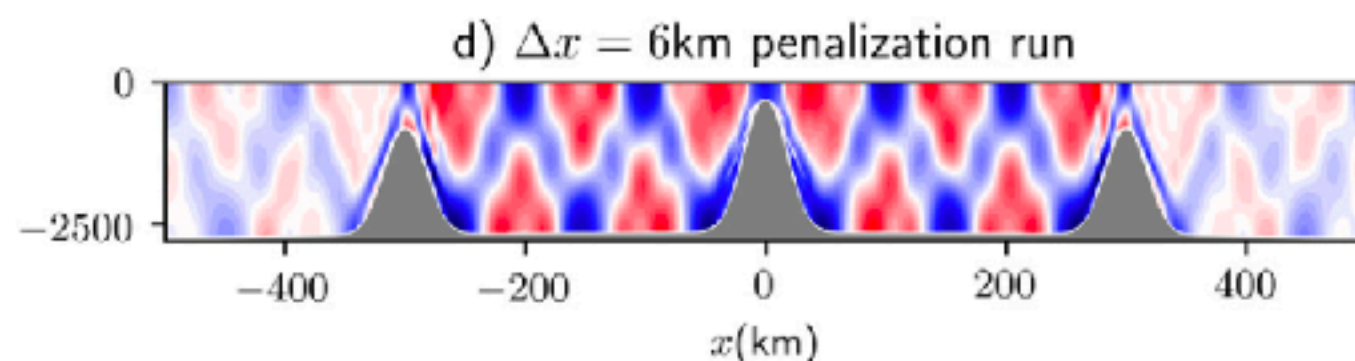
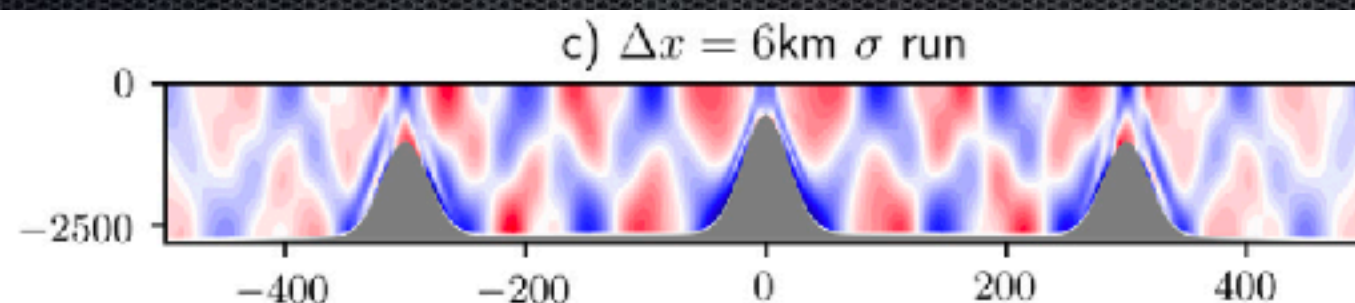
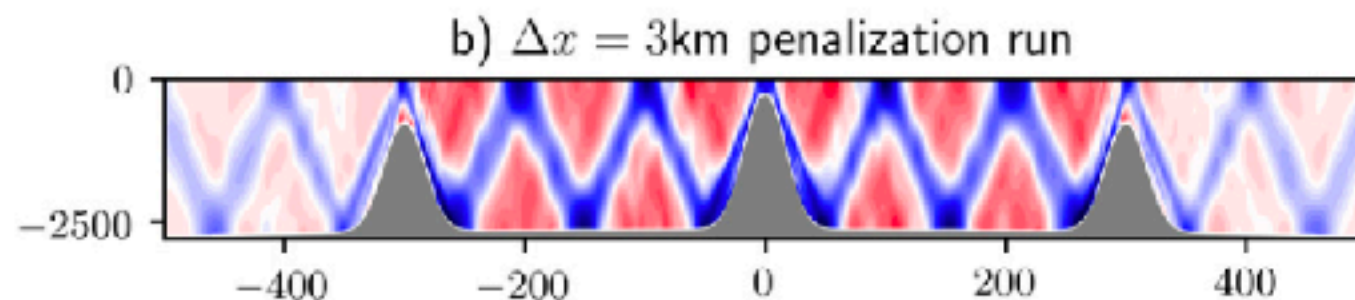
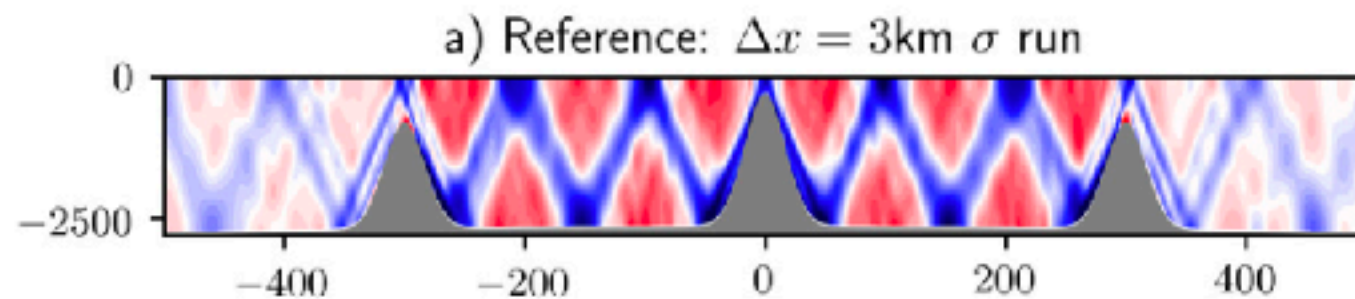
u (m/s)



r limited

high-resolution

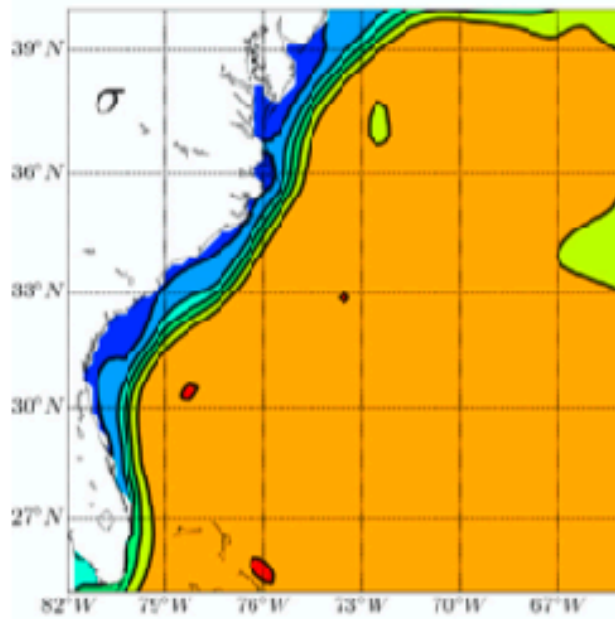
low-resolution: the penalized run retains much of the high-resolution energy



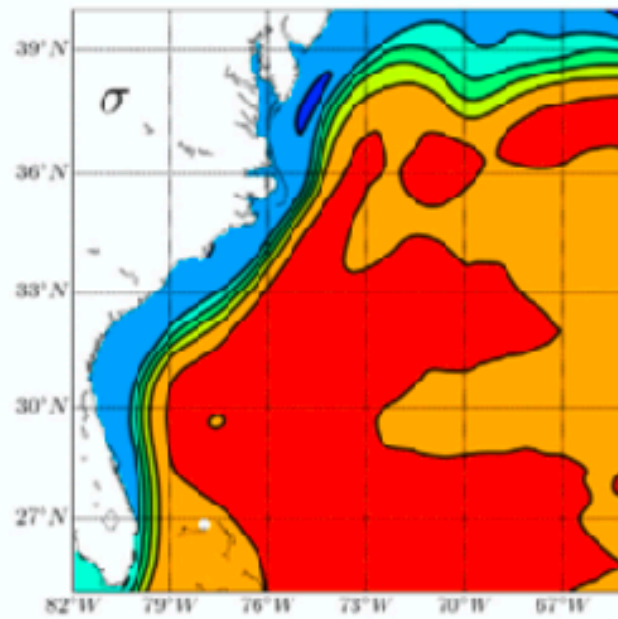
Gulf Stream penalization

Resolution sensitivity

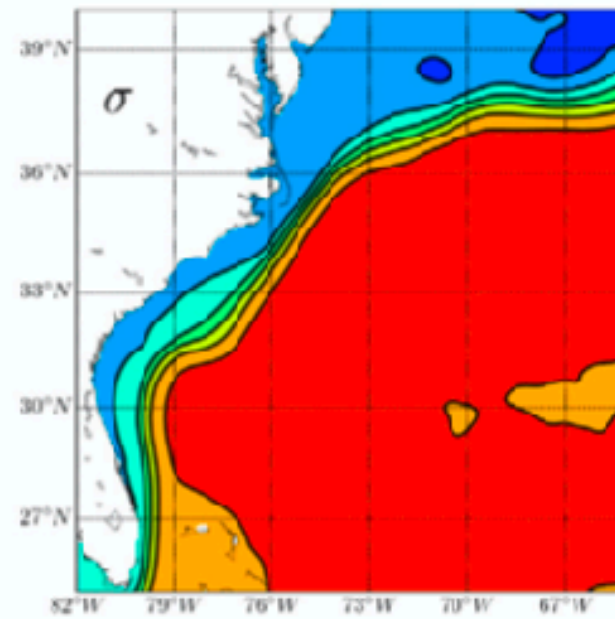
$1/4^\circ$



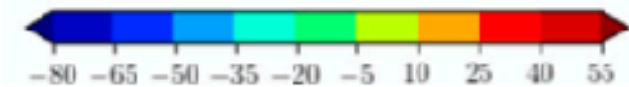
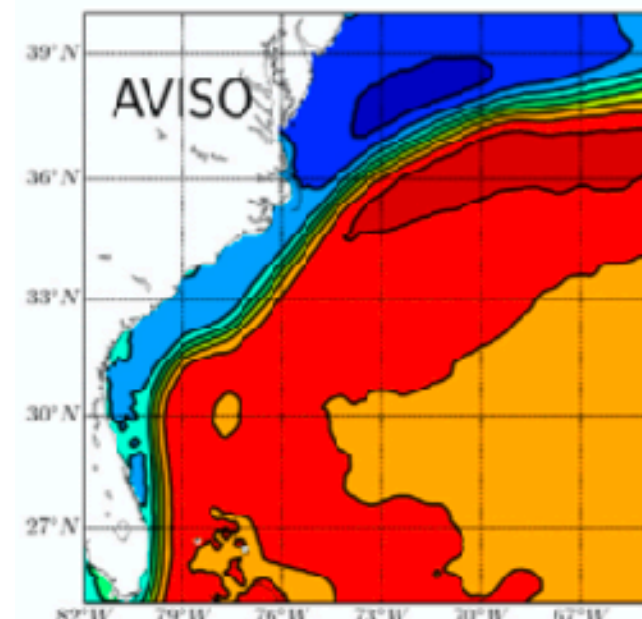
$1/8^\circ$



$1/12^\circ$

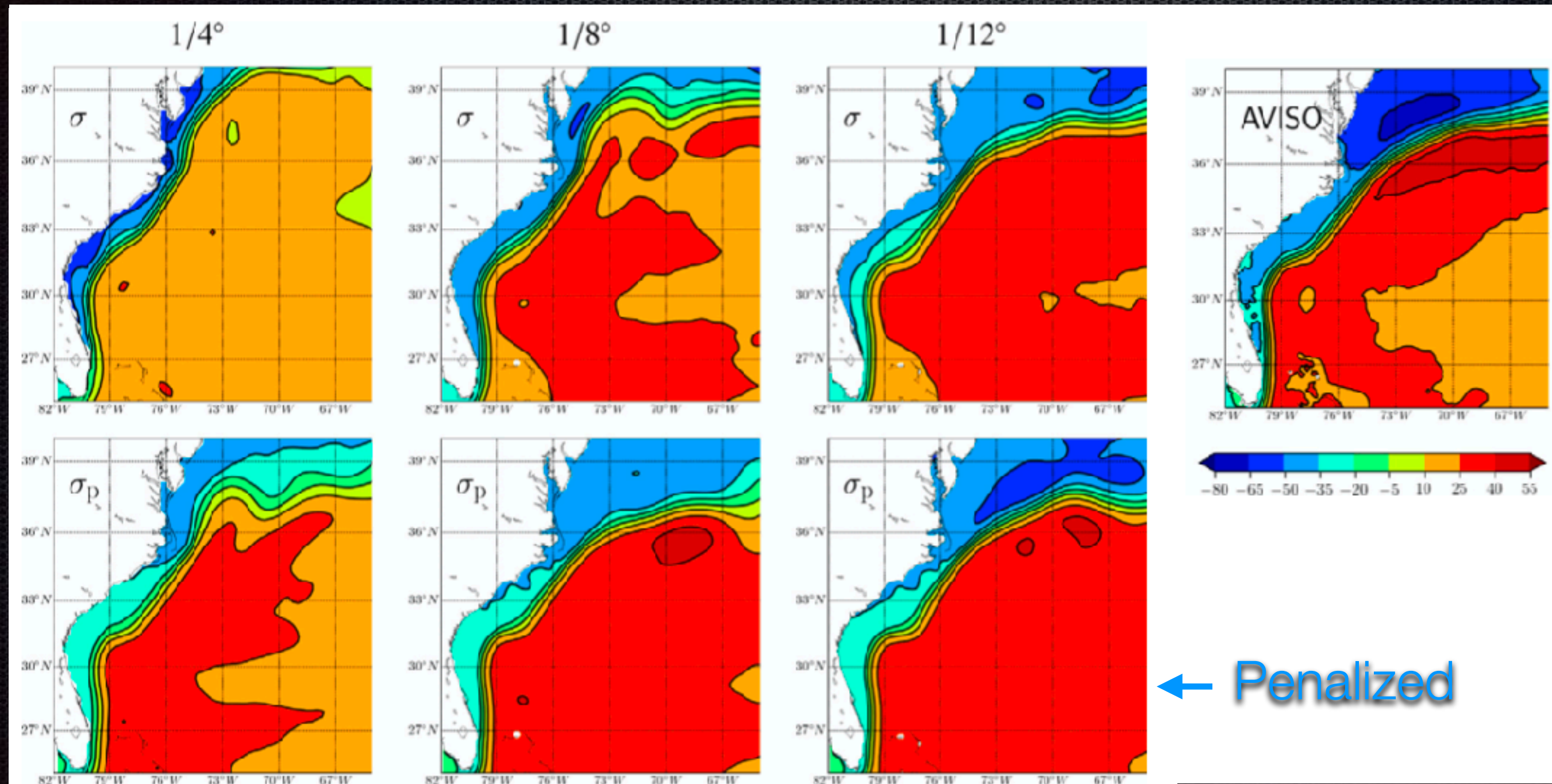


AVISO



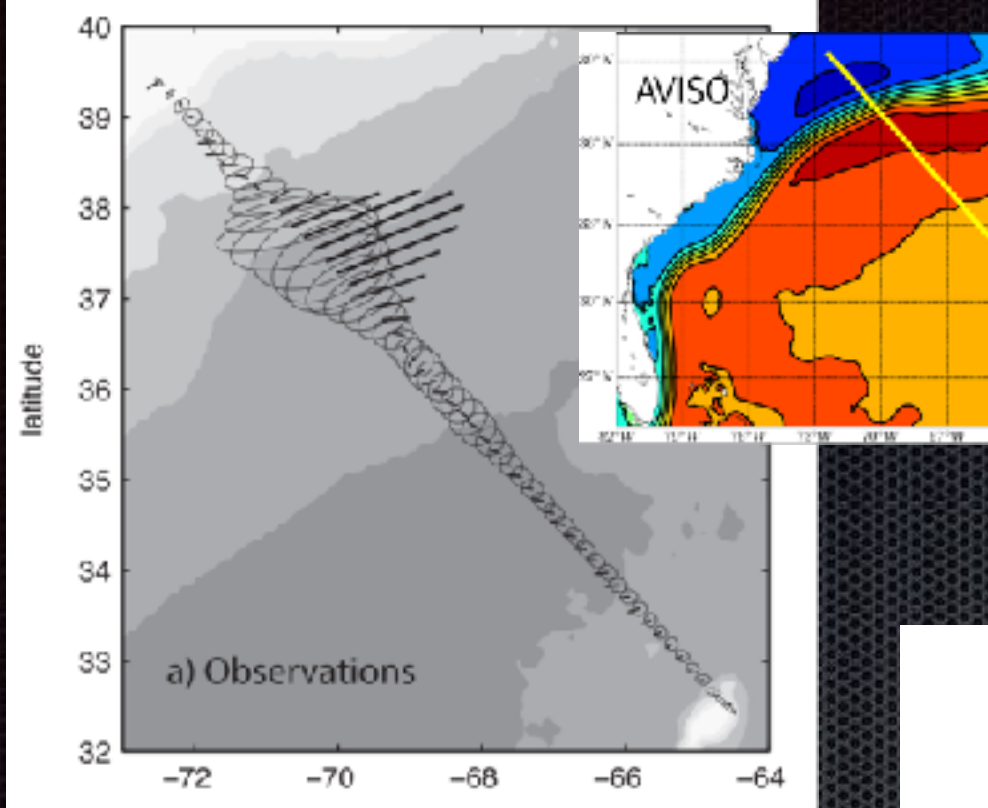
Gulf Stream penalization

Resolution sensitivity

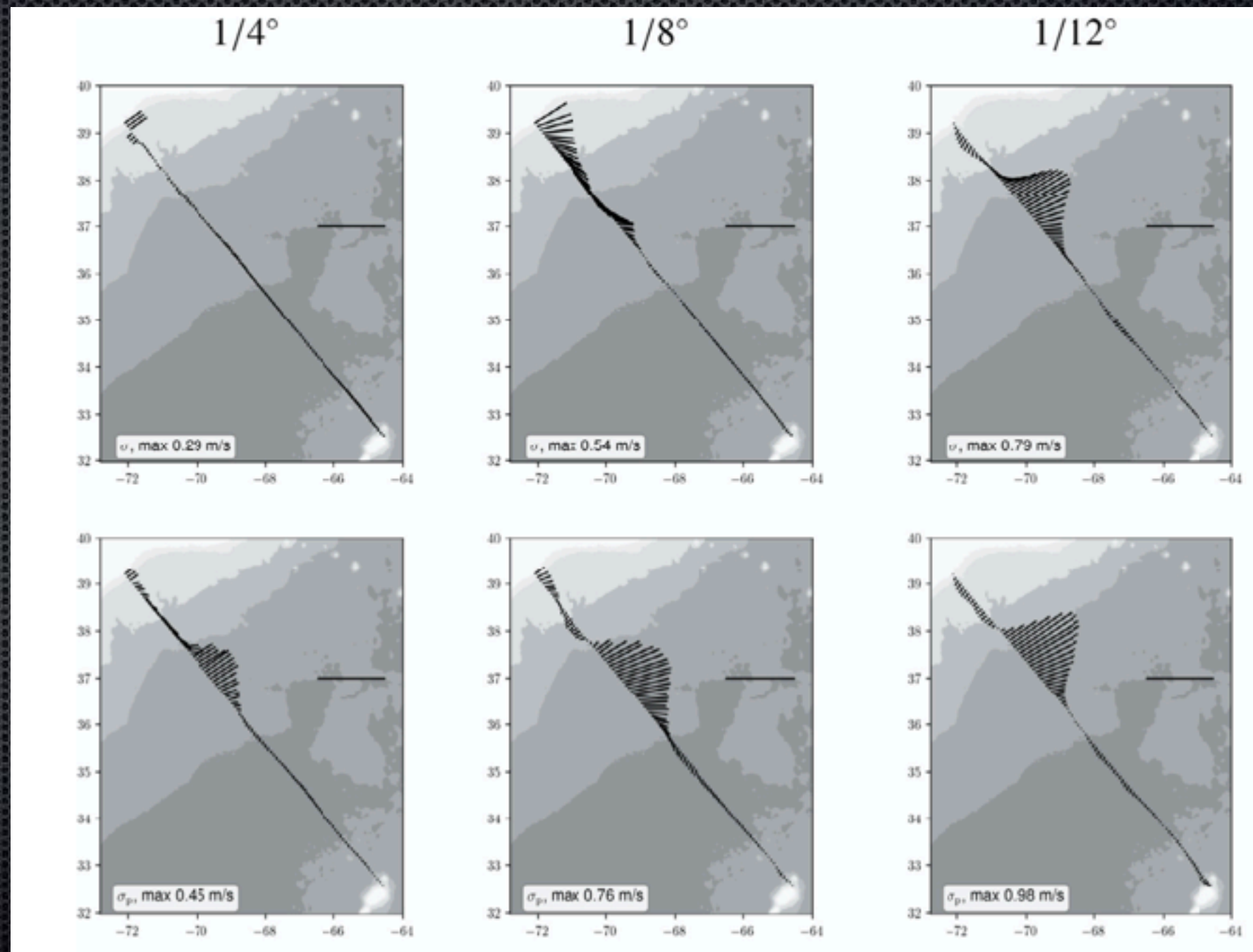


Comparison with in-situ data

Oleander ADCP transect
(1993-2012)



sigma



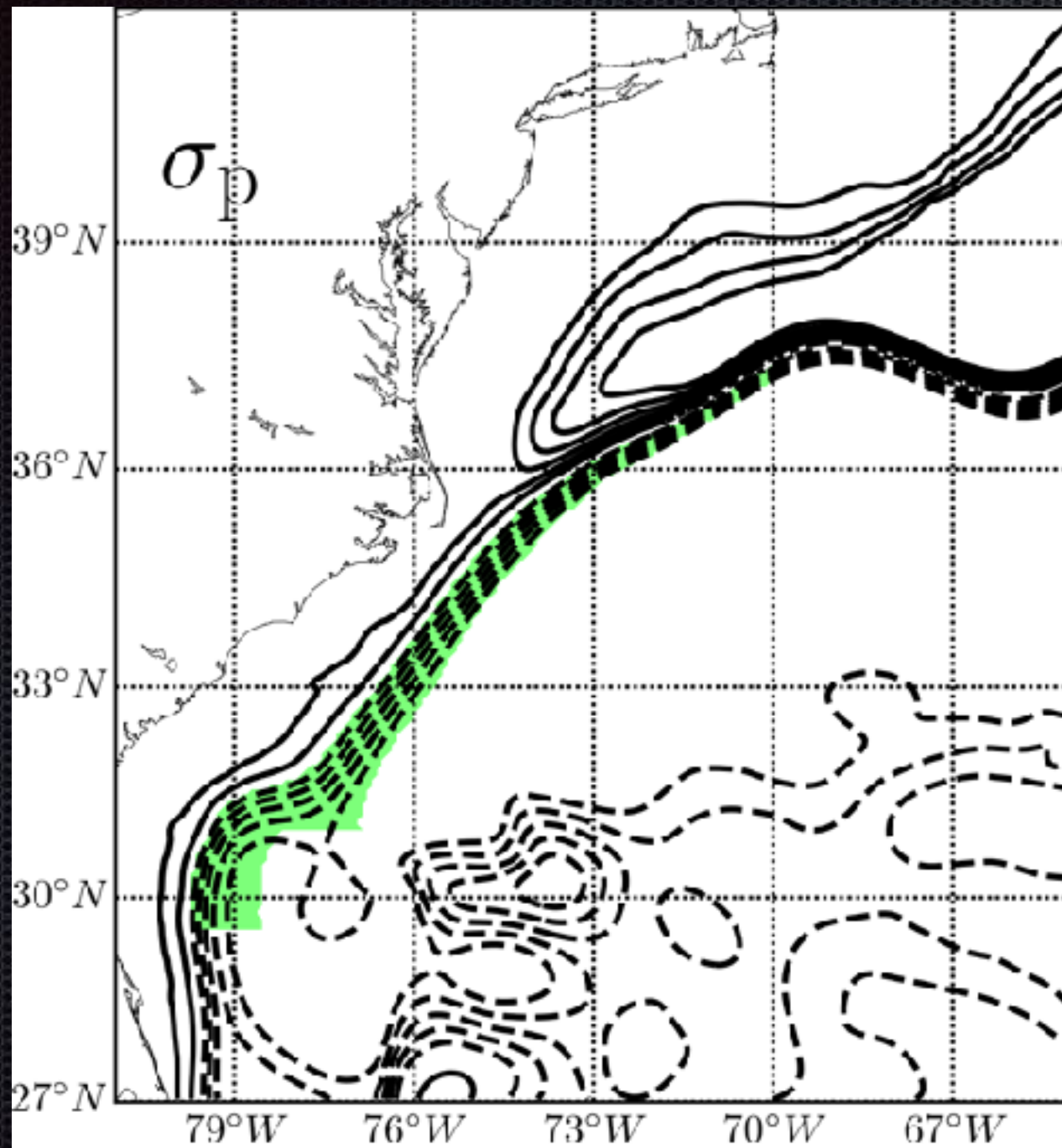
penalized

Barotropic Vorticity Budget

$$\frac{\partial \bar{\zeta}}{\partial t} = \underbrace{\frac{J(P_b, h)}{\rho_0}}_{\text{Bottom Pressure Torque}} - \underbrace{\nabla \cdot (f\mathbf{U})}_{\text{Planetary Vorticity Advection}} - \underbrace{A}_{\text{Advection Torque (eddy activity)}} + \underbrace{\frac{\nabla \times \boldsymbol{\tau}}{\rho_0}}_{\text{Wind Stress Curl}} - \underbrace{\frac{\nabla \times \boldsymbol{\tau}_b}{\rho_0}}_{\text{Bottom Stress Curl}} + \underbrace{D}_{\text{Viscous Torque}}$$

$\bar{\zeta} = (\nabla \times \mathbf{U}) \cdot \hat{\mathbf{z}}$ is the barotropic vorticity

GS Barotropic Vorticity Budget

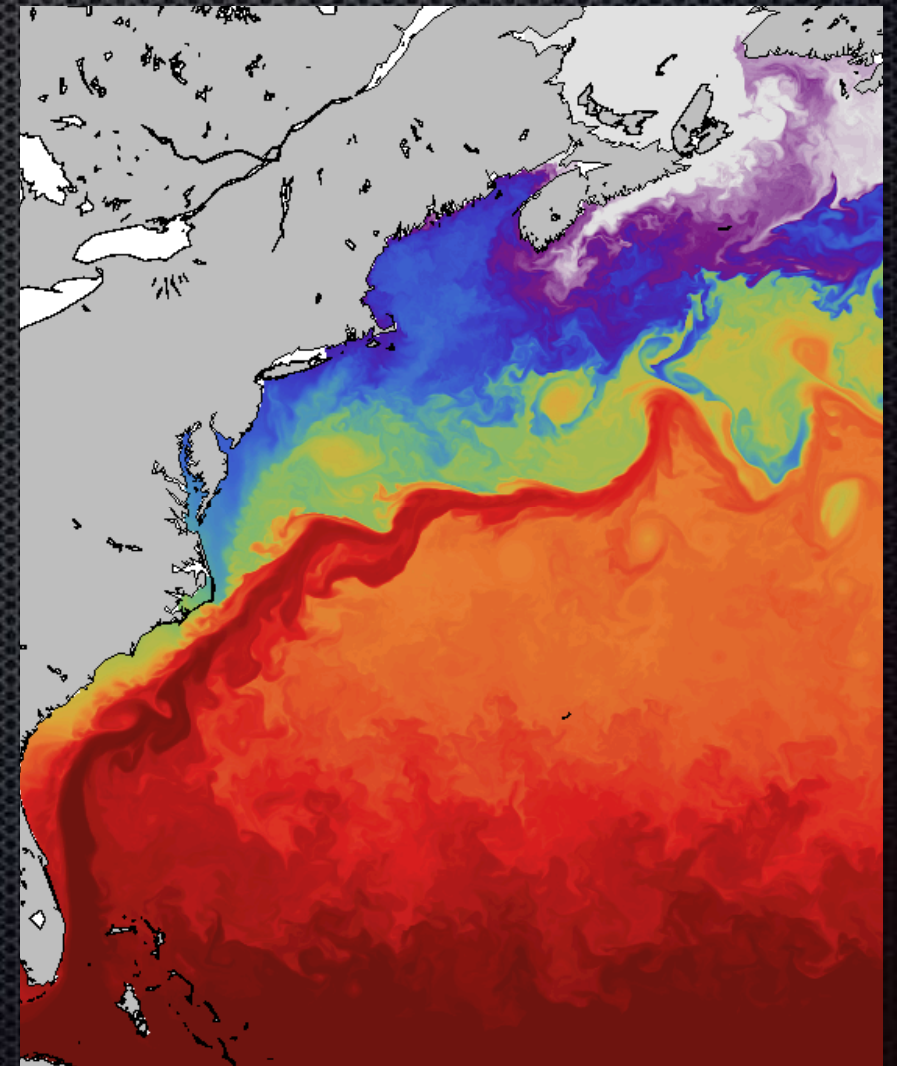


budget integrated between
1 and 30 Sv contours
(Schoonover et al., 2016)

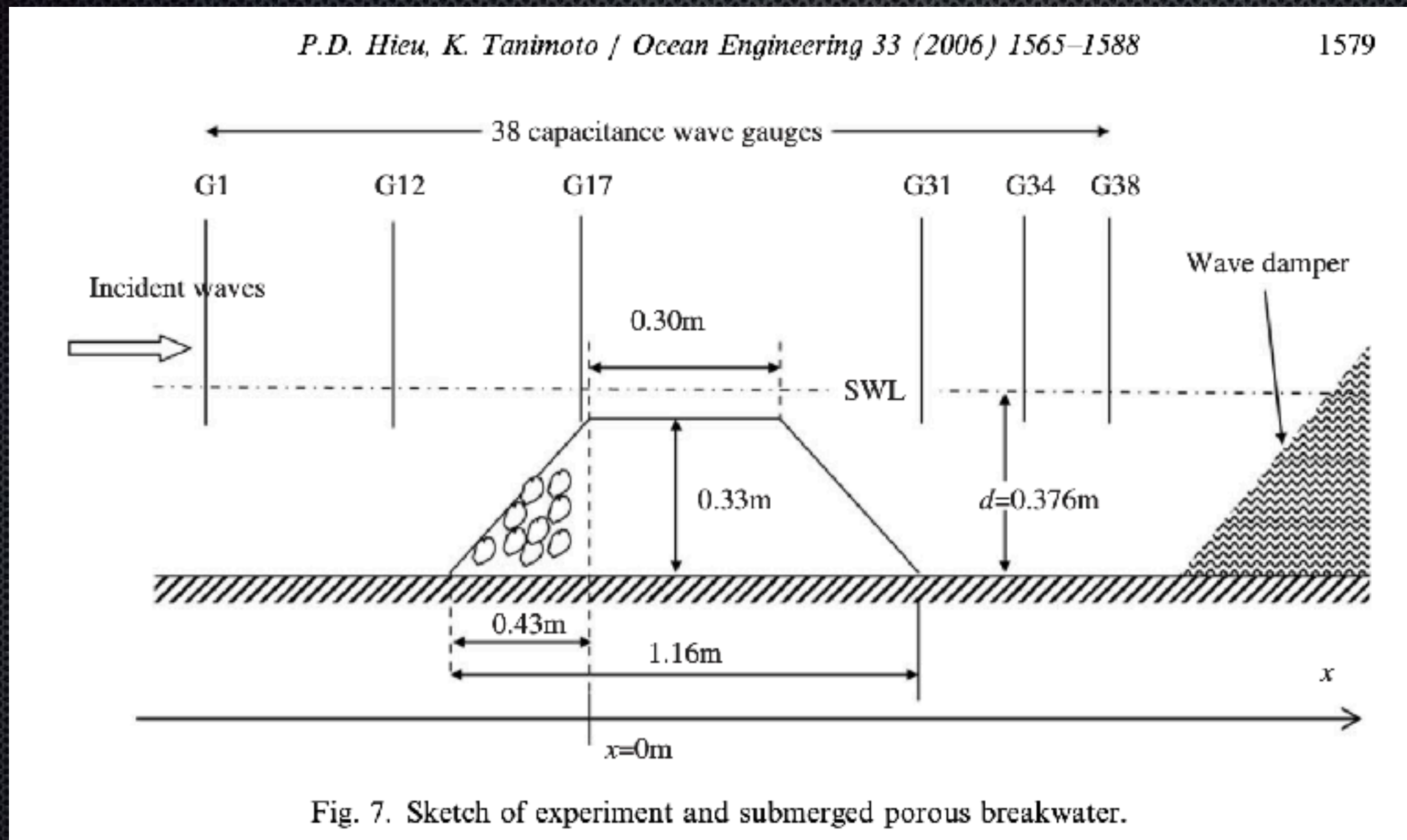
GS Barotropic Vorticity Budget

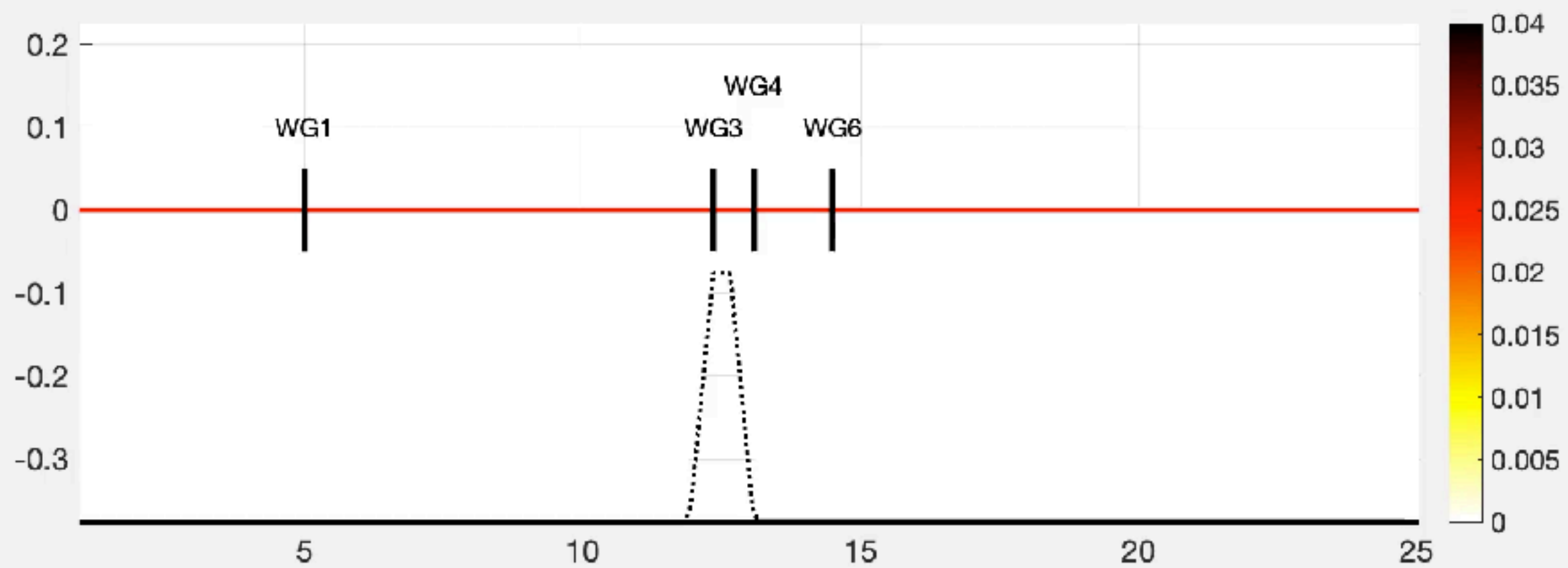
steering *inertia* *eddies*

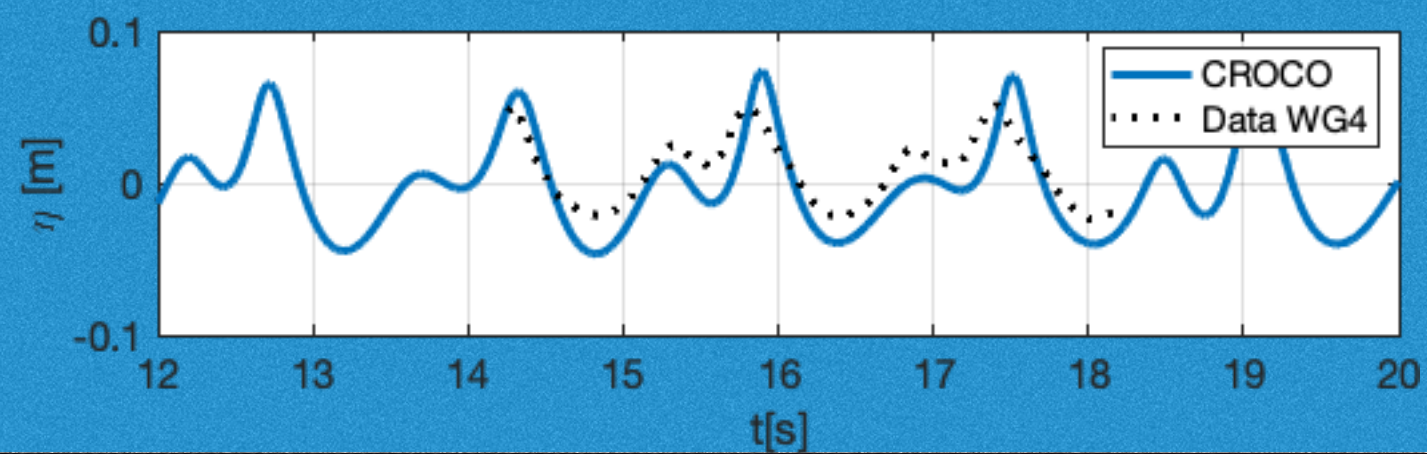
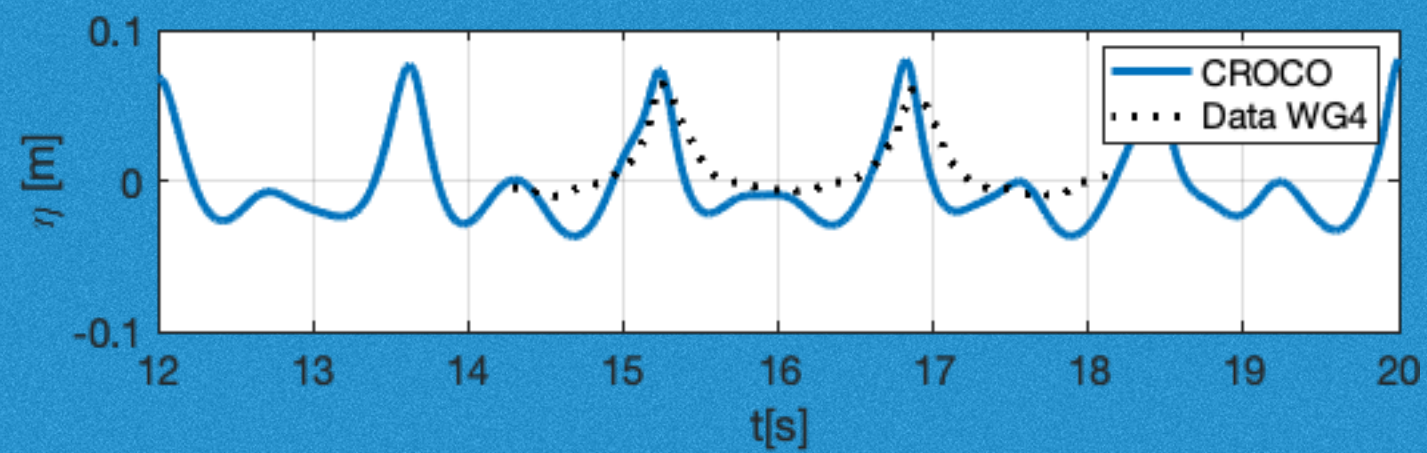
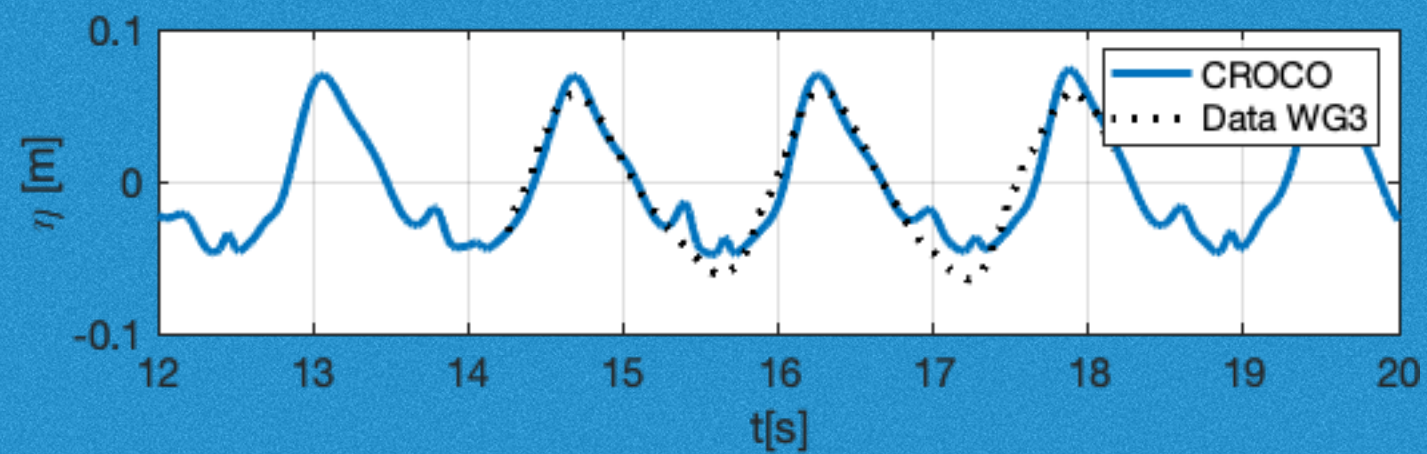
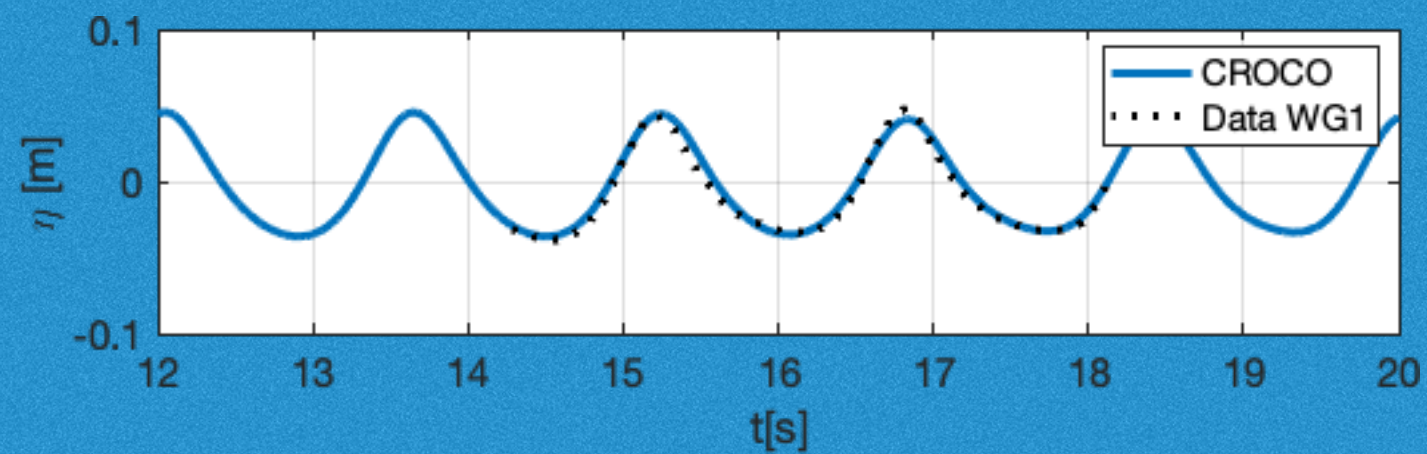
	BPT	BETA	ADV
$\sigma \ 1/8^\circ$	0.93	-1.78	0.54
$\sigma \ 1/12^\circ$	1.83	-2.48	0.29
pena $1/8^\circ$	1.77	-2.65	0.26
pena $1/12^\circ$	2.24	-2.75	0.04



NBQ : Test porous breakwater







Conclusion

- * A simple and efficient penalization method is implemented in a **working branch** of CROCO
 - ✓ First immersed BCs in a realistic ocean model
- * GS results relevant to **climate modeling**
- * Many other possible applications
 - ✓ Subgrid bathymetry, vegetation, coastal protection, urban flooding

