CROCO

Coastal and Regional Ocean COmmunity model

CROCO-NBQ applications

fine-scale non-hydrostatic dynamics

CROCO trainings, Brest 2019

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Non-hydrostatic solver: advances

- Compressible (NBQ) approach (Auclair et al., 2017)
- Pressure correction method (Roullet, Molemaker, Ducousso)
  - Merging proposed in spring
- Capabilities recovered in NH:
  - OBC, Wet/Dry, WCI, 1-way nesting (?), land proc elimination(?)
  - All realistic and idealized configurations can be done in NBQ
- Capabilities that are still missing
  - OpenMP, 2-way nesting, fast-step diffusion (myalpha?)
- Physical/numerical closure
  - SGS models: 3D Smagorinsky / GLS
  - Monotone (shock-capturing) schemes (WENO5-Z, TVD)

Non-hydrostatic solver: algorithm

- Pressure correction method

Homogeneous linearized equations

$$
\begin{aligned}
\partial_x u &+ \partial_z w = 0 \\
\partial_t u &= -g\partial_x \eta - \partial_x q/\rho_0 \\
\partial_t w &= -\partial_z q/\rho_0 \\
\\
\partial_t \eta &= w(0) = -H\partial_x \overline{u}
\end{aligned}
$$

Split-explicit algorithm $0 \leq m \leq N_{\mathrm{split}} - 1$

1. Advance η and $\overline{u}$ with $\overline{q}^{\star} = 0$ or $\overline{q}^{\star} = \overline{q}^{n}$

$$
\begin{cases}
\overline{u}^{m+1} = \overline{u}^{m} - g(\delta t)\partial_x \eta^{m} - \frac{\delta t}{\rho_0}\partial_x \overline{q}^{\star} \\
\eta^{m+1} = \eta^{m} - \delta t H \partial_x \overline{u}^{m+1},
\end{cases}
$$

2. Compute provisional fields $\widetilde{u}^{n+1}$ and $\widetilde{w}^{n+1}$
3. Correct $\widetilde{u}^{n+1}$ to enforce $\overline{\widetilde{u}^{n+1}} = \overline{u}^{n+1}$
4. Solve $\Delta q = \dfrac{\rho_0}{\Delta t}\left(\partial_x \widetilde{u}^{n+1} + \partial_z \widetilde{w}^{n+1}\right)$
5. Correct velocity field to remove divergent part

$$
u^{n+1} = \widetilde{u}^{n+1} - \Delta t \partial_x q, \quad w^{n+1} = \widetilde{w}^{n+1} - \Delta t \partial_z q
$$

However : $\overline{u}^{n+1} \neq \overline{u^{n+1}}$

Solution : change boundary condition on q to $\partial_z q|_{z=0} = 0$
$\Rightarrow \overline{u}^{n+1} = \overline{\widetilde{u}^{n+1}} = \overline{u^{n+1}}$

+ Discard of barotropic non-hydrostatic mode
+ Complexity of solving Poisson equation in sigma coordinates
+ Scalability issues

Non-hydrostatic solver: algorithm

- Pressure correction method
- Compressible approach (Auclair et al., 2017)

$$p = p_a + p_H + c_s^2 \delta\rho$$

Homogeneous linearized equations

$$
\begin{aligned}
\partial_t u &= -g\partial_x \eta - c_s^2 \partial_x \delta\rho \\
\partial_t w &= -c_s^2 \partial_z \delta\rho \\
\partial_t \delta\rho &= -\rho_0 (\partial_x u + \partial_z w) \\
\\
\partial_t \eta &= w|_{z=0} \\
w|_{z=-H} &= 0 \\
\delta\rho|_{z=0} &= 0
\end{aligned}
$$

Acoustic mode integrated in a split-explicit free surface approach at the same fast step as the barotropic mode

Semi-implicit forward-backward

$$
\begin{aligned}
u^{m+1} &= u^m - \delta t \left(g\partial_x \eta^m + c_s^2 \partial_x \delta\rho^m \right) \\
w^{m+1} &= w^m - \delta t c_s^2 \partial_z \left(\delta\rho^{m+\theta} \right) \\
\delta\rho^{m+1} &= \delta\rho^m - \rho_0 \delta t \left(\partial_x u^{m+1} + \partial_z w^{m+\theta} \right) \\
\eta^{m+1} &= \eta^m + \delta t (w|_{z=0})^{m+\theta}
\end{aligned}
$$

Non-hydrostatic solver: algorithm

- Pressure correction method
- Compressible approach (Auclair et al., 2017)

Physics

- Solves short surface waves
- Solves mixed acoustic-gravity waves (tsunami precursor)
- High-order pressure gradient → accuracy for internal waves

Performances

- Same fast step as hydrostatic code because of :
  - ✓ possible reduction of c_S (> $\sqrt{gh}$)
  - ✓ semi-implicit treatment
- Scalability: scales well with resolution

COST NH ~ 3 x H

Step3d_fast

https://www.overleaf.com/read/jcpqcjgmvyqp

http://poc.omp.obs-mip.fr/auclair/WOcean.fr/SNH/index_snh_home.htm

```
!
!
!******************************************************************
!
!                  SOLVE FAST MODE 3D EQUATIONS
!
!******************************************************************
!
!  This routines:
!  1- Computes non-NBQ RHS forcing terms of momentum equations. First
!     computes the barotropic (external) RHS forcing term (rubar,rvbar)
!     then adds it to the internal RHS forcing (computed in pre_step3d).
!  2- Solves the 3D momentum conservation equations for fast-mode
!     components (qdmu_nbq, qdmv_nbq, qdmw_nbq) by time integration of
!     all forces:
!       Compressible pressure force + second viscosity + gravity
!       + NT Coriolis force + restoring force + non-NBQ RHS forces
!  3- Solves mass conservation equation, i.e., computes compressible
!     density rho_nbq by time integration of momentum divergence
!
!  In this version, a first guest of zeta is derived from the surface
!  vertical velocity (surface characteristic relation) instead of the
!  depth-averaged conservation of mass. This satisfies dynamical coupling
!  with the surface layer. After solving the 3D momentum equations, a
!  final zeta field is diagnozed from mass conservation (then Hz is also
!  corrected for the internal time step).
!
!  W-momentum equation is solved with explicit or implicit methods:
!  - Explicit scheme: w-momentum is updated right after (and the same
!                     way as) u- and v-momentum.
!  - Implicit scheme: horizontal component of divergence is first
!                     precomputed (as required by fast-mode mass
!                     conservation) before tridiagonal Gauss Elimination
!                     is carried out for qdmw_nbq(m).
!
!  For all components, a Forward-Backward scheme is implemented:
!  - Explicit scheme: Forward:  zeta, qdmu_nbq, qdmw_nbq.
!                     Backward:       rho_nbq.
!  - Implicit scheme: Forward:  zeta, qdmu_nbq.
!                     Backward:       qdmw_nbq, rho_nbq.
!
!  In the NBQ_PERF option, the vertical grid is not evolving at fast
!  time step to gain computational time.
!
!******************************************************************
!
```

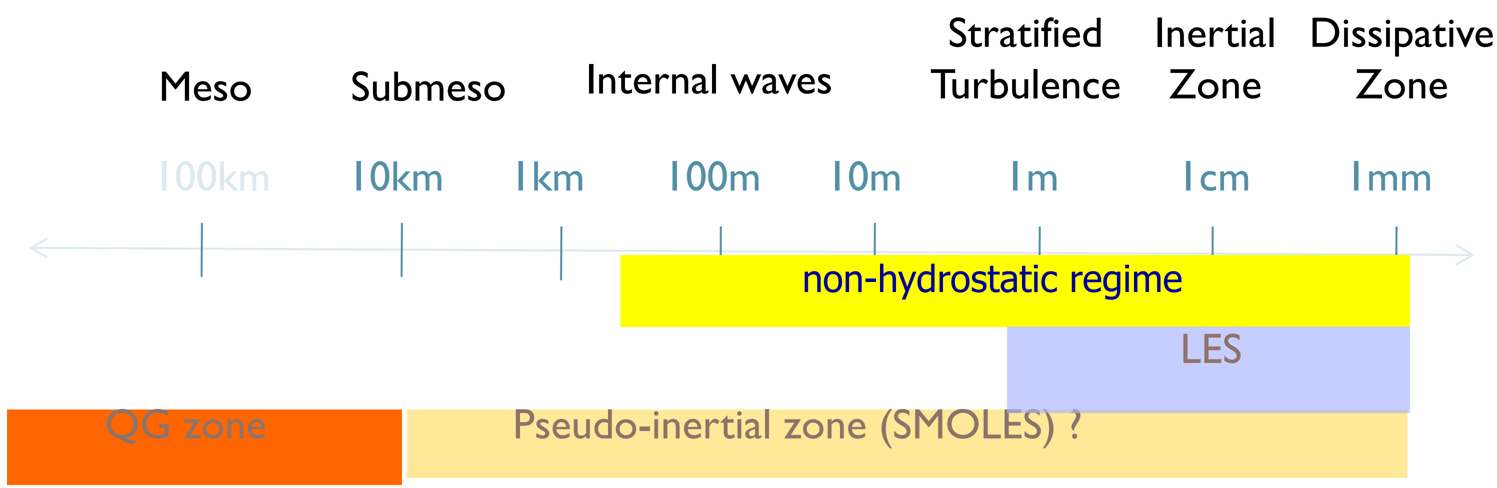
Semi-implicit forward-backward

$$
\begin{aligned}
u^{m+1} &= u^m - \delta t\left(g\partial_x\eta^m + c_s^2\partial_x\delta\rho^m\right)\\
w^{m+1} &= w^m - \delta t c_s^2\partial_z\left(\delta\rho^{m+\theta}\right)\\
\delta\rho^{m+1} &= \delta\rho^m - \rho_0\delta t\left(\partial_x u^{m+1} + \partial_z w^{m+\theta}\right)\\
\eta^{m+1} &= \eta^m + \delta t(w|_{z=0})^{m+\theta}
\end{aligned}
$$

Applications

Submesoscale dynamics
Internal bores
Breaking internal tides
Turbulence mixing
Surface wave dynamics
River plumes

Energy flows in a realistic, coupled ocean and in geographically heterogeneous regimes



Quasi-hydrostatic equations
non-traditional Coriolis terms

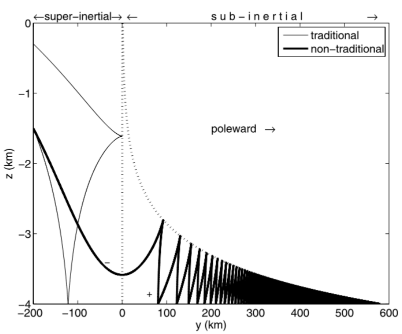
Bertrand Delormes, Stanford U.

Marshall et al., 1997; Gerkema et al, 2008

$$\frac{\partial u}{\partial t} + (\mathbf{V}_3 \cdot \nabla)u - fv + f^* w + \frac{\partial \phi}{\partial x} - \mu_{\mathbf{v}} \Delta_h u - \nu_{\mathbf{v}} \frac{\partial^2 u}{\partial z^2} = 0,$$

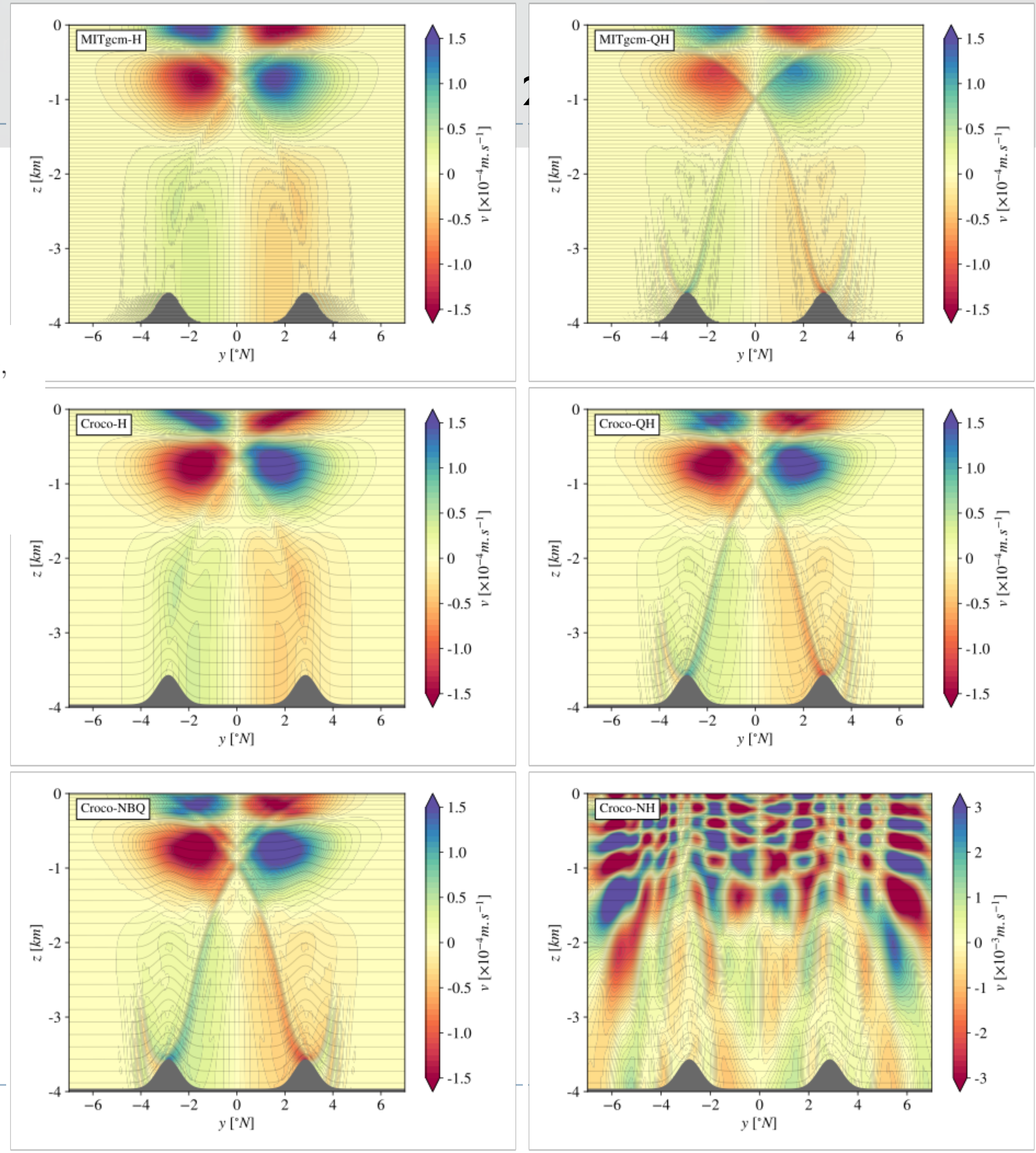
$$\frac{\partial v}{\partial t} + (\mathbf{V}_3 \cdot \nabla)v + fu + \frac{\partial \phi}{\partial y} - \mu_{\mathbf{v}} \Delta_h v - \nu_{\mathbf{v}} \frac{\partial^2 v}{\partial z^2} = 0,$$

$$- f^* u + \frac{\partial \phi}{\partial z} = -\frac{\rho}{\rho_0} g,$$

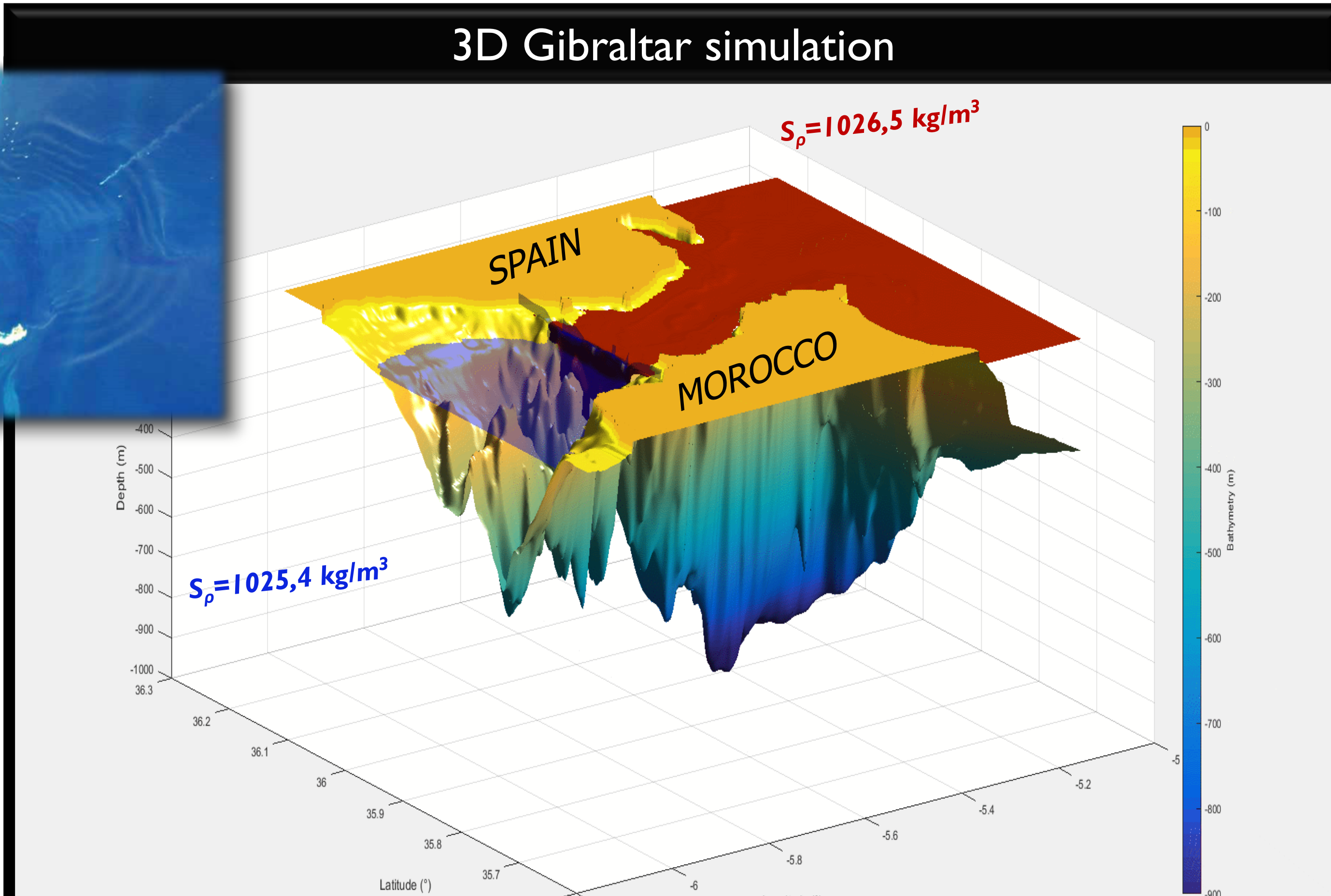


Are NT terms essentially fast modes?

Equatorial Wave Over Topography
150 days into the simulation (wave period = 10 days)
Bertrand Delorme – 10/10/2018

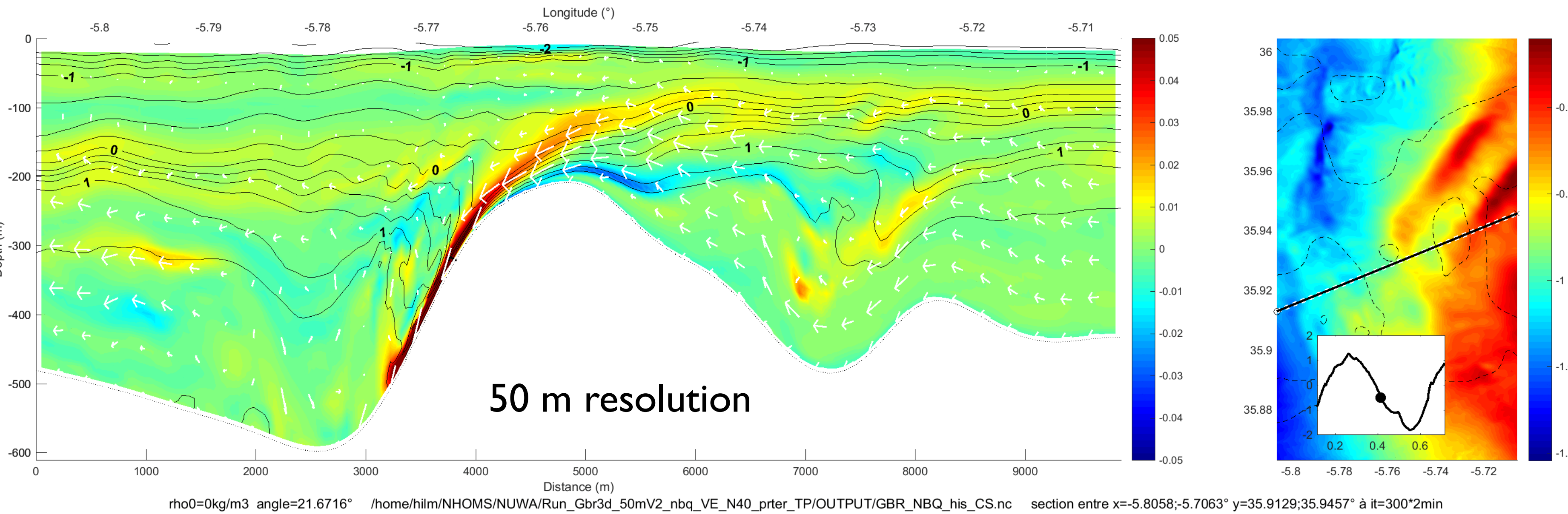


Nonlinear internal waves



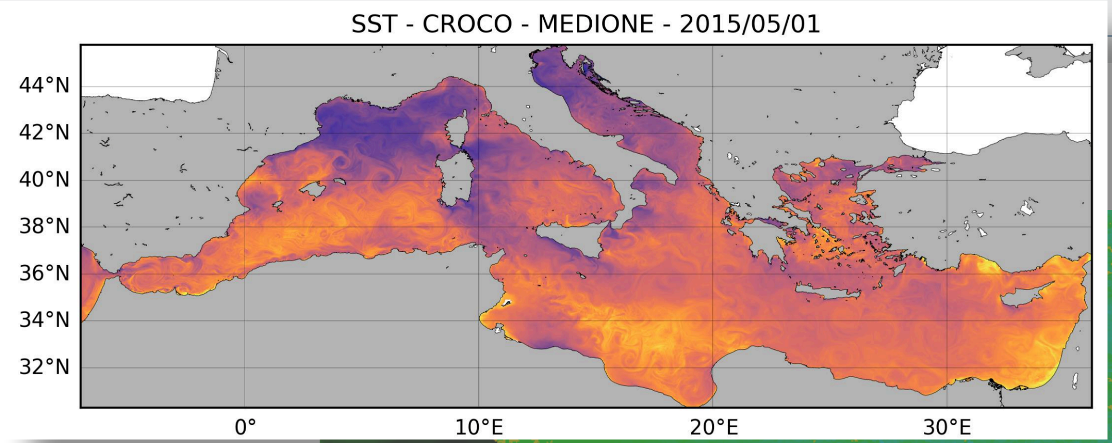
Bordois et al., 2018

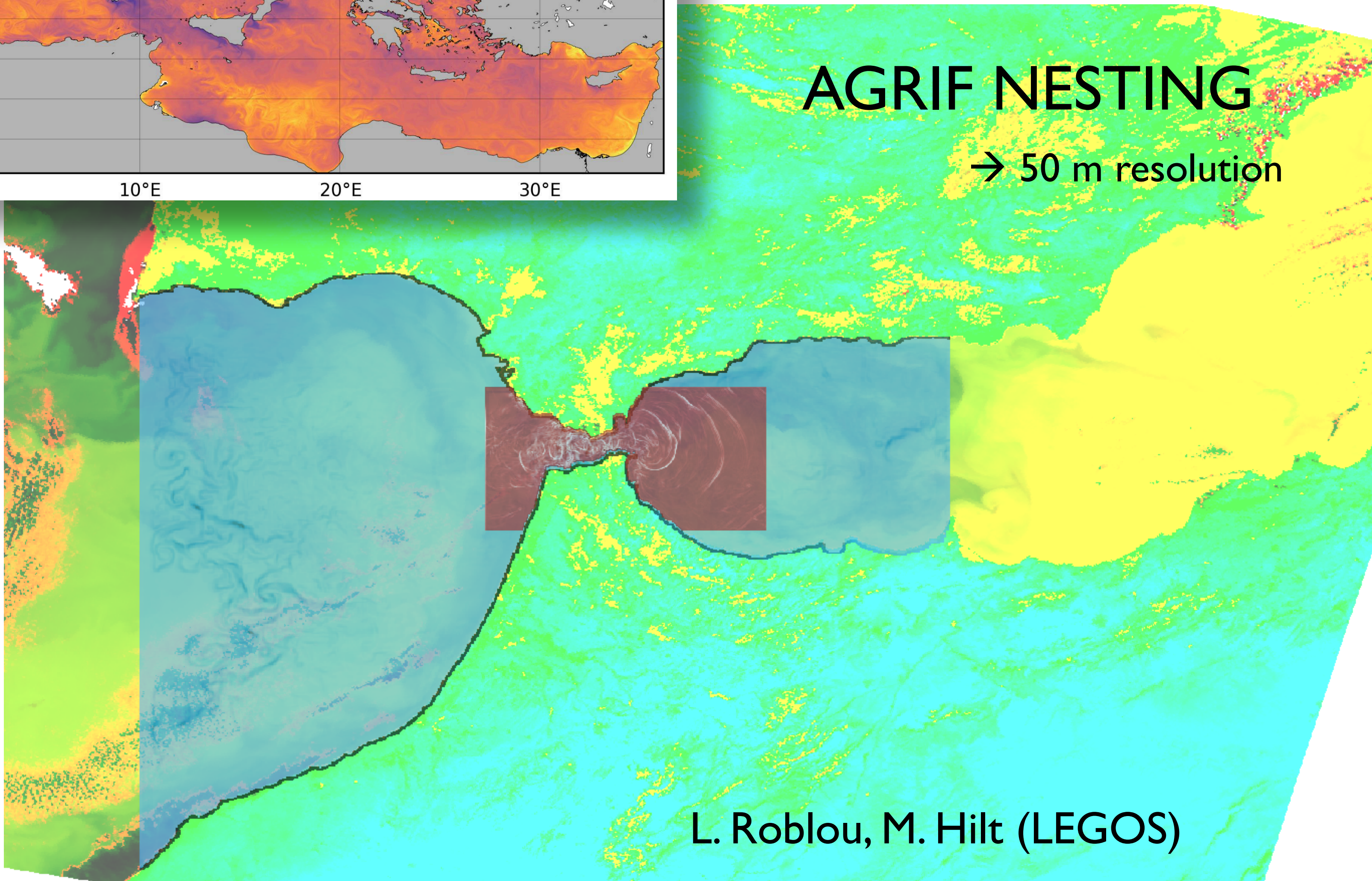
Internal hydraulic jump



PhD Margaux Hilt, 2019

Nesting: Gibraltar / Mediterranean

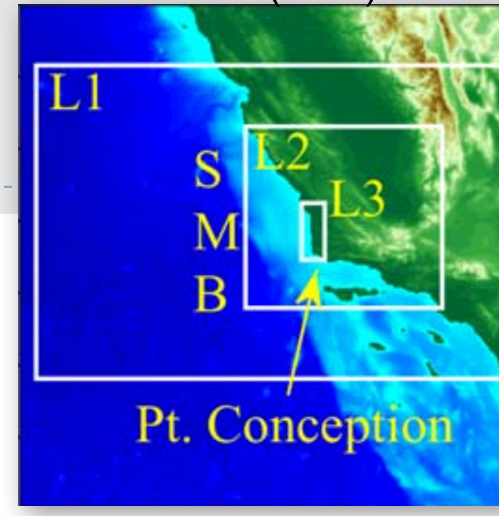


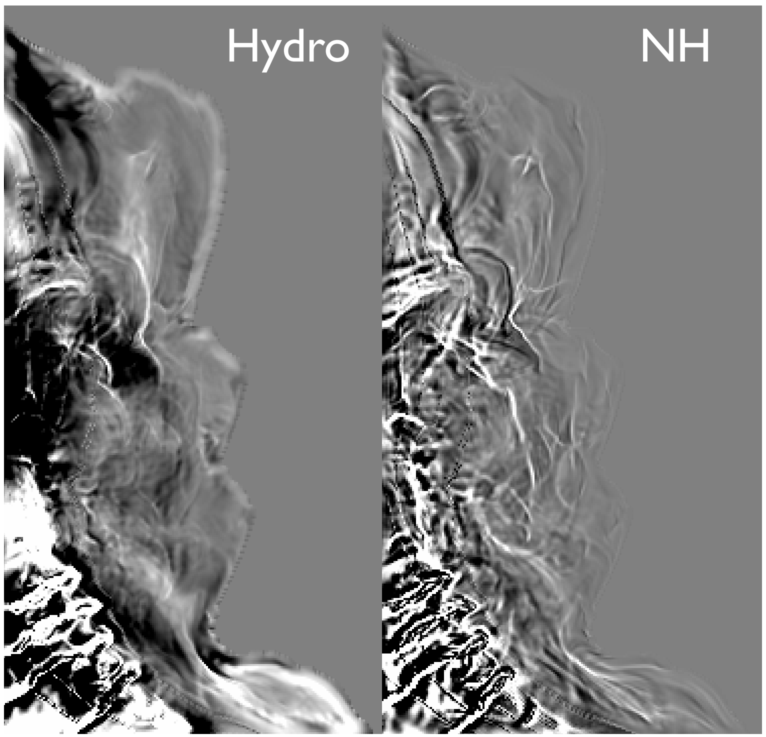


Coastal internal wave breaking

Kumar et al., UW, Seattle

Suanda et al. (2017)





Bottom W

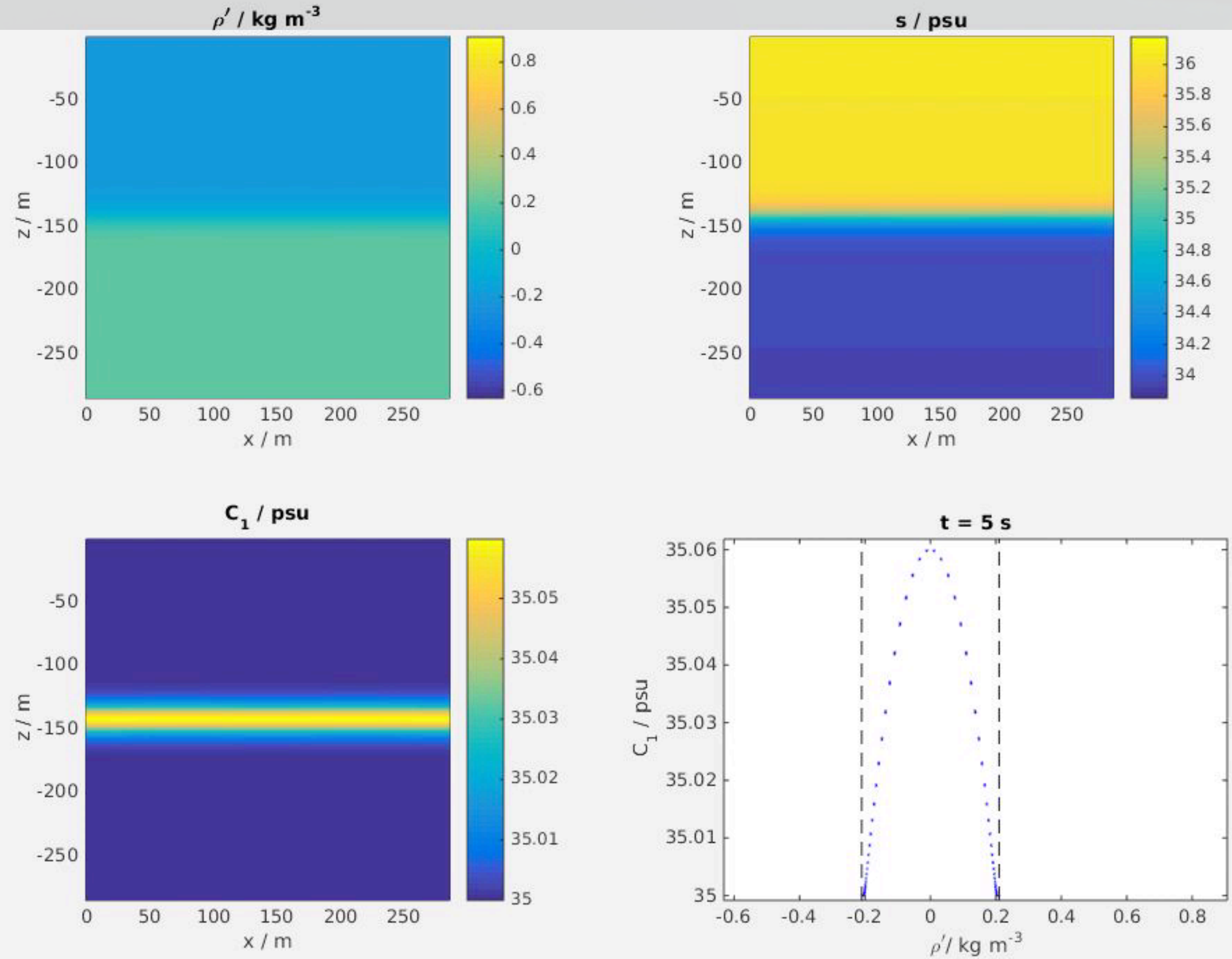
Hydro

NH

3D Turbulence mixing

TEASEO project
(Y. Morel, P. Haynes)

Penney et al., 2018



KH instability - 1 m resolution

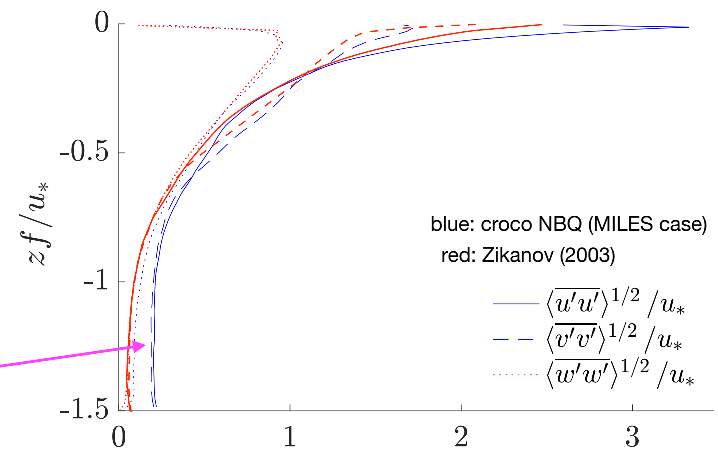
Ekman layer LES

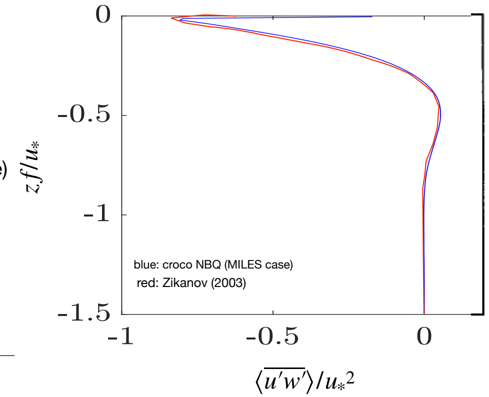
Nobuhiro Suzuki

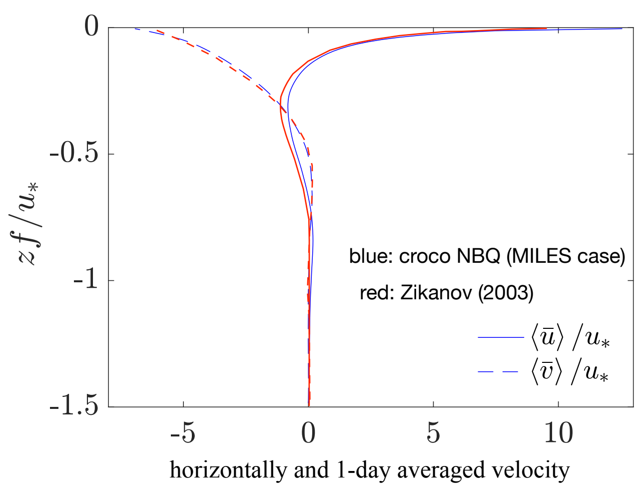
Comparison with
Zikanov (2003),
J. Fluid Mech., 495, pp. 343–368.

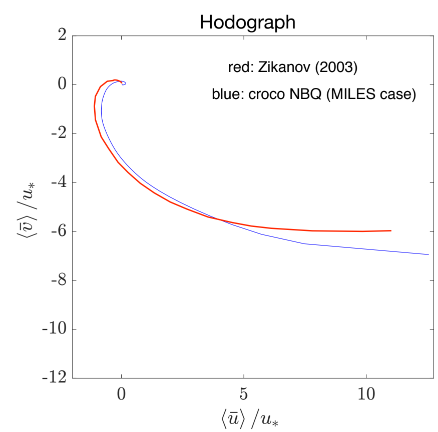
Configuration
- periodic horiz. bound. cond.
- free-slip bottom
- constant stress at the top
- domain: 1 x 1 x 1.5 u*/ f
- resolution:
  48 x 48 x 90 nodes for Croco NBQ
  64 x 64 x 120 nodes for Zikanov (2003)

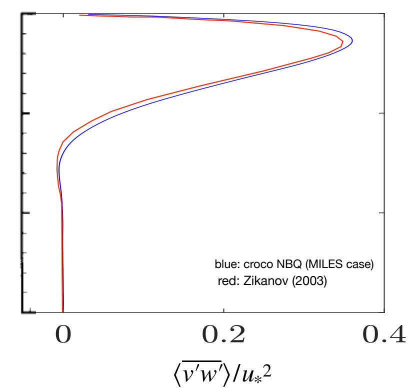
In Croco NBQ, u' & v' occur even below the Ekman layer. (i.e., zf/u* < -1)





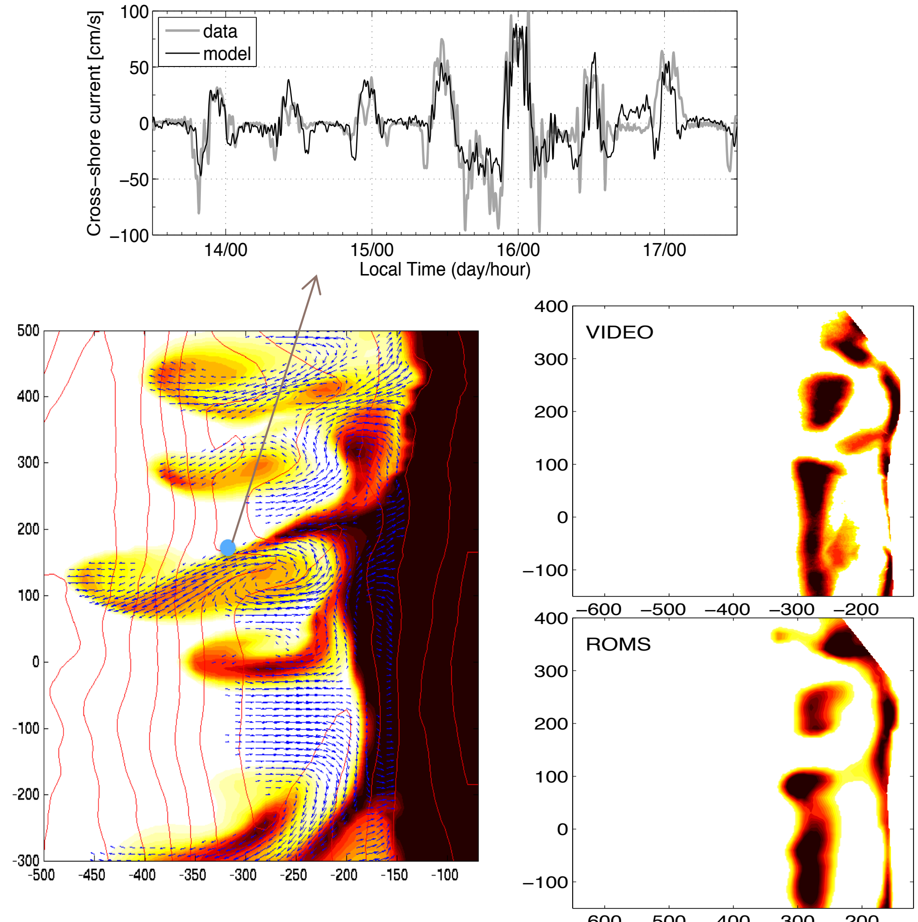






WAVE-AVERAGED MODELING

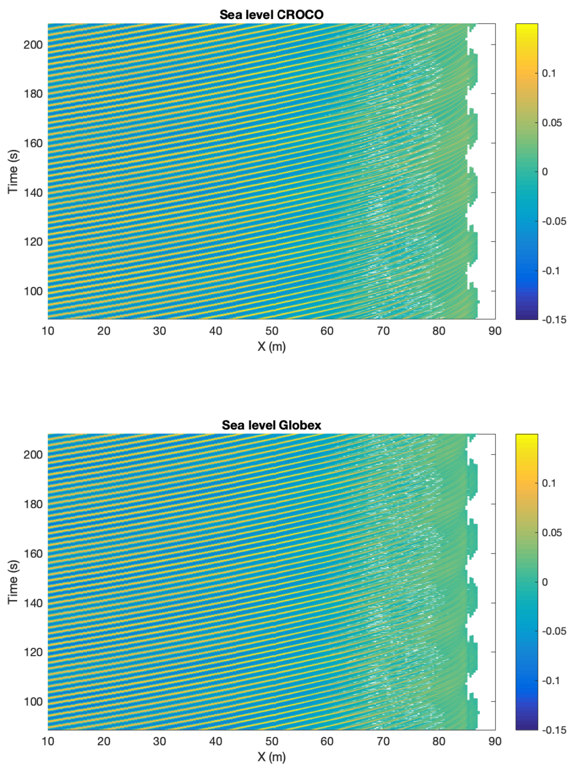
Now works in NBQ mode
(Wetting/Drying, Stokes drift)



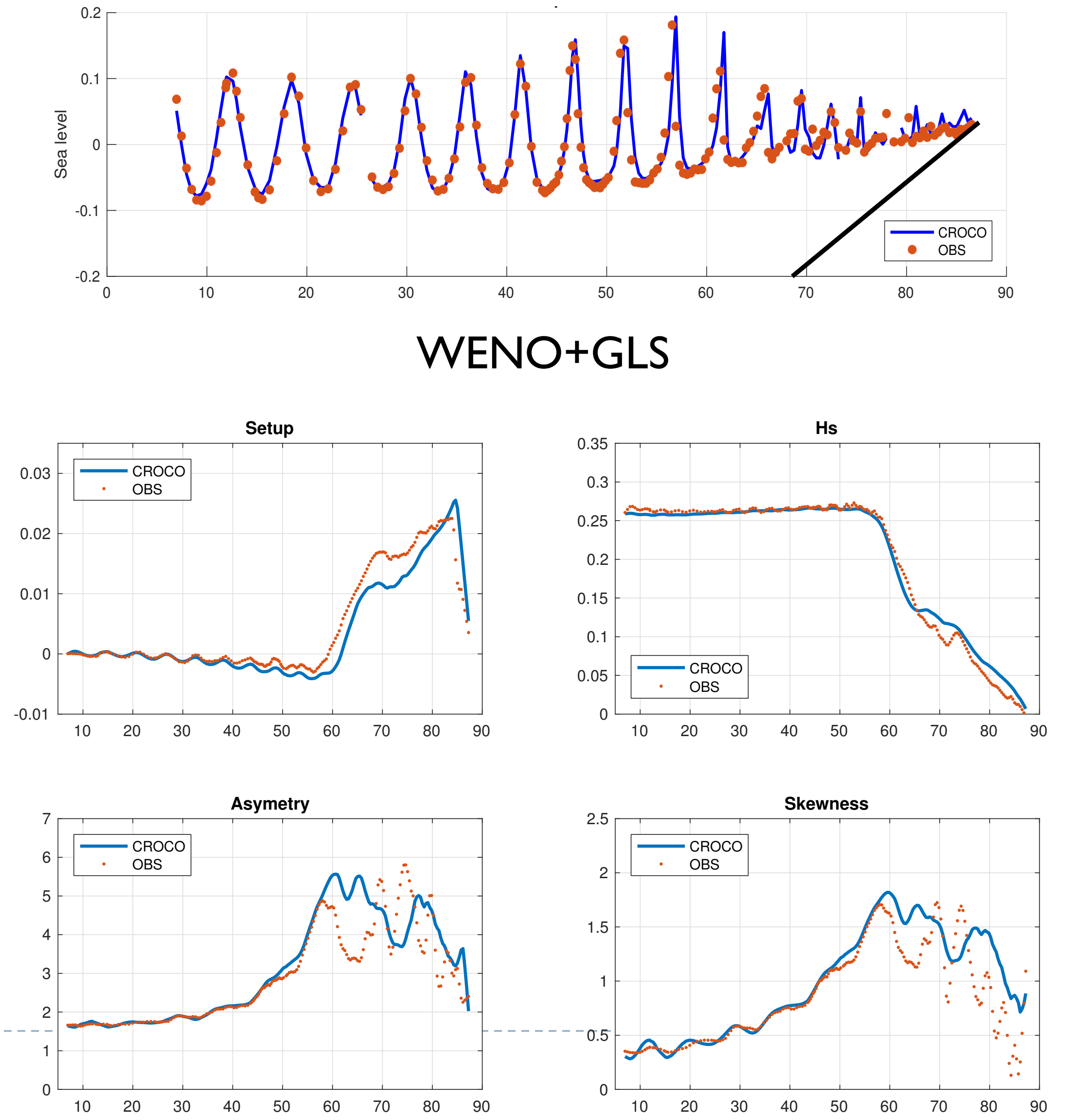
Marchesiello et al. (2015)

WAVE-RESOLVED MODELING

Validation of wave breaking with flume experiment (GLOBEX)

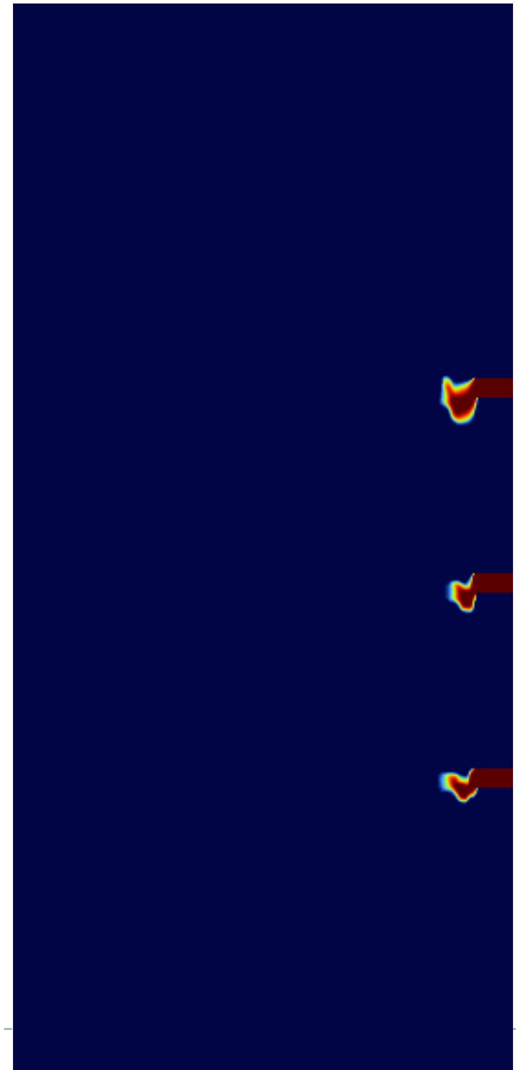


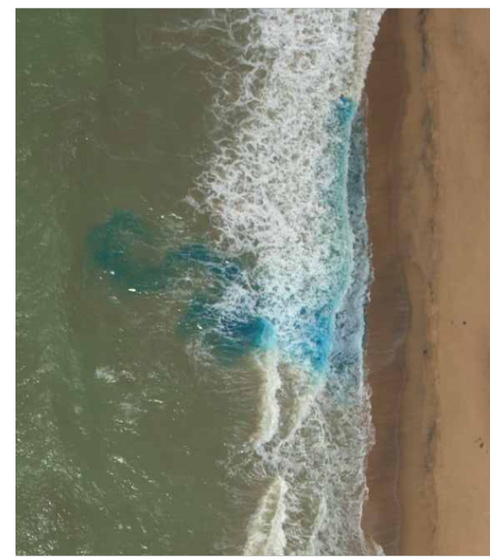
Cs=1500 m/s
NBQ_PRECISE



WAVE-RESOLVED MODELING

`Wave_maker.h`



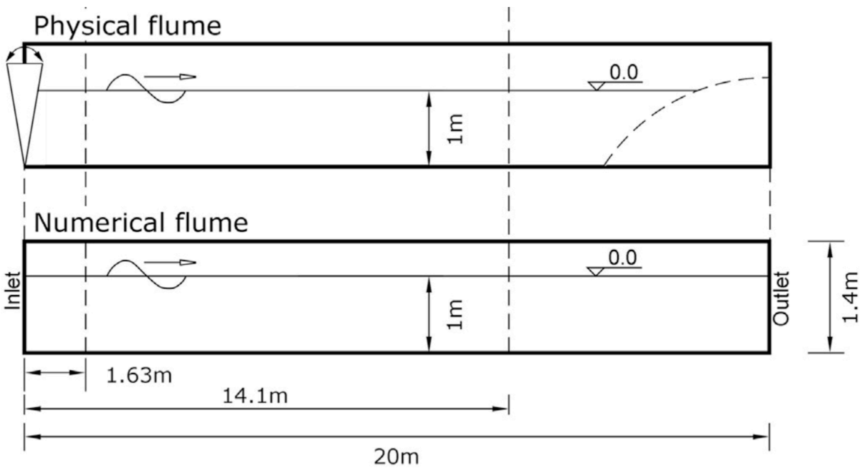


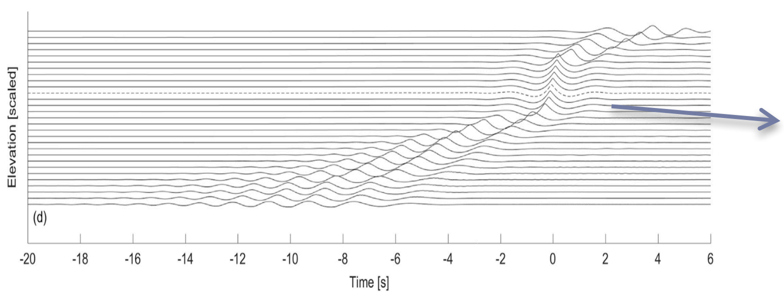
Dye exp. Grand Popo, Benin

Rogues waves

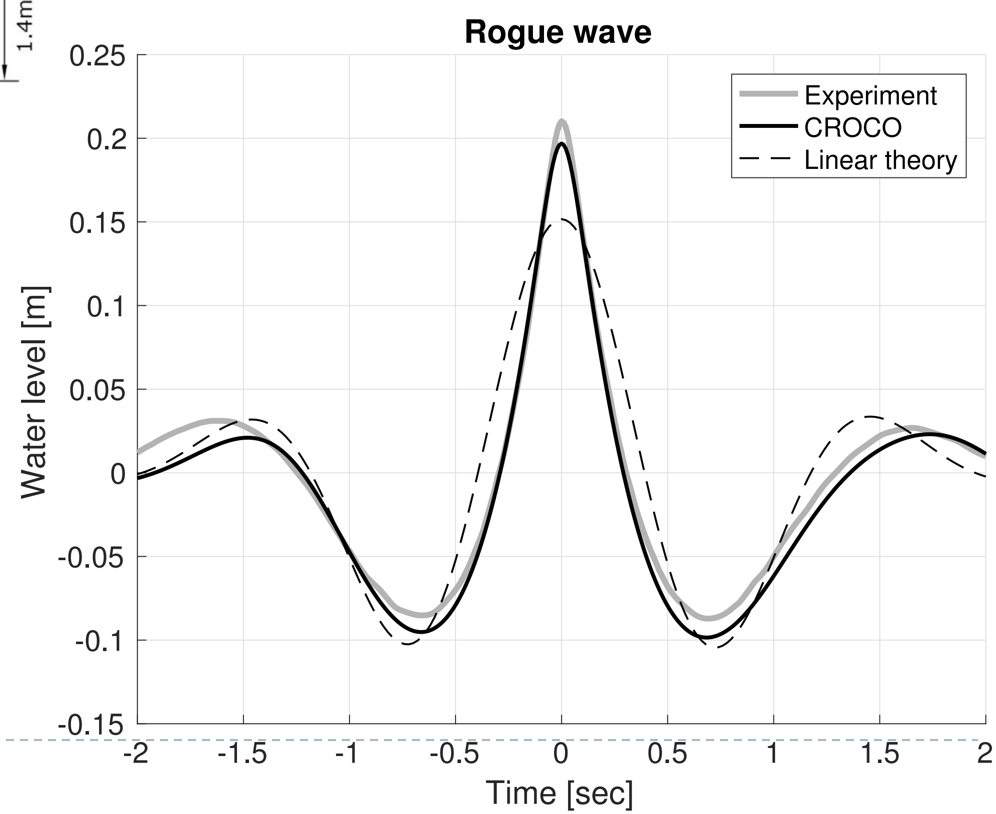
Vysikas et al., 2018

Generation of focused wave group





Shape and elevation not predicted by linear or 2nd order wave theory. This is an effect of high order non-linearities in large transient waves



Role of high-order non-linearity and mixed acoustic-gravity waves ?

Numerical methods

Physical closure (LES vs MILES)

- **CROCO SGS models :**
  - ✓ 3D Smagorinsky
  - ✓ 3D GLS

$$K_M = c\sqrt{2k}\,l S_M + \nu,$$

$$\frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial z}\left(\frac{K_M}{\sigma_k}\frac{\partial k}{\partial z}\right) + P + B - \varepsilon$$

Shear (3D) Buoyancy (homog. case)

- **Physical/numerical closure**

$\nu_{Smag} \sim C_S\, L\, V$ $\qquad C_S \sim 0.01$

$\nu_{Num} \sim C_N\, L\, V$ $\qquad C_N = 1/12$ (UP3)
$\qquad 1/60$ (UP5)

To function, SGS models must be associated with advection schemes that reduce numerical diffusion and include shock-capturing (MILES)