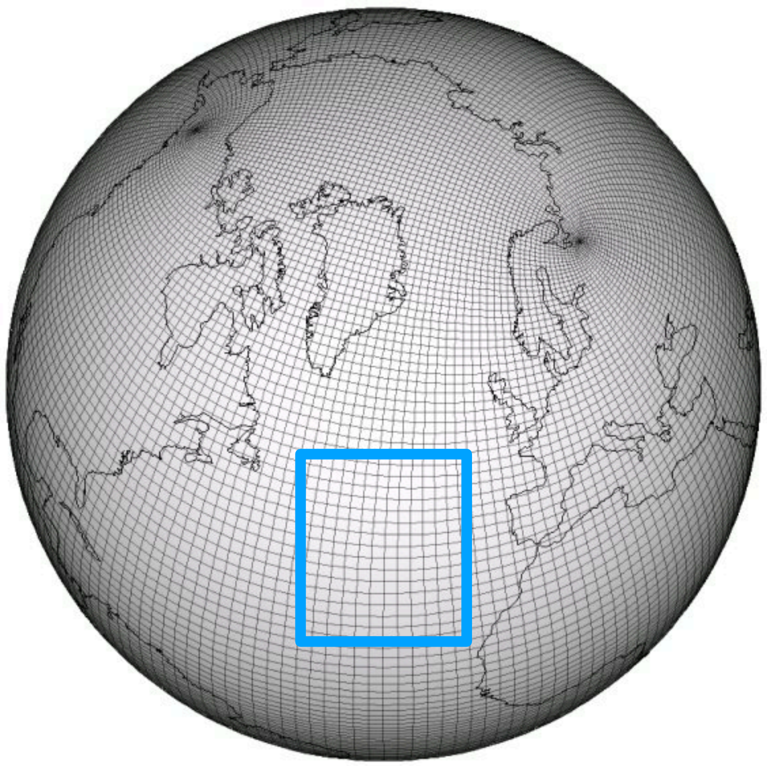
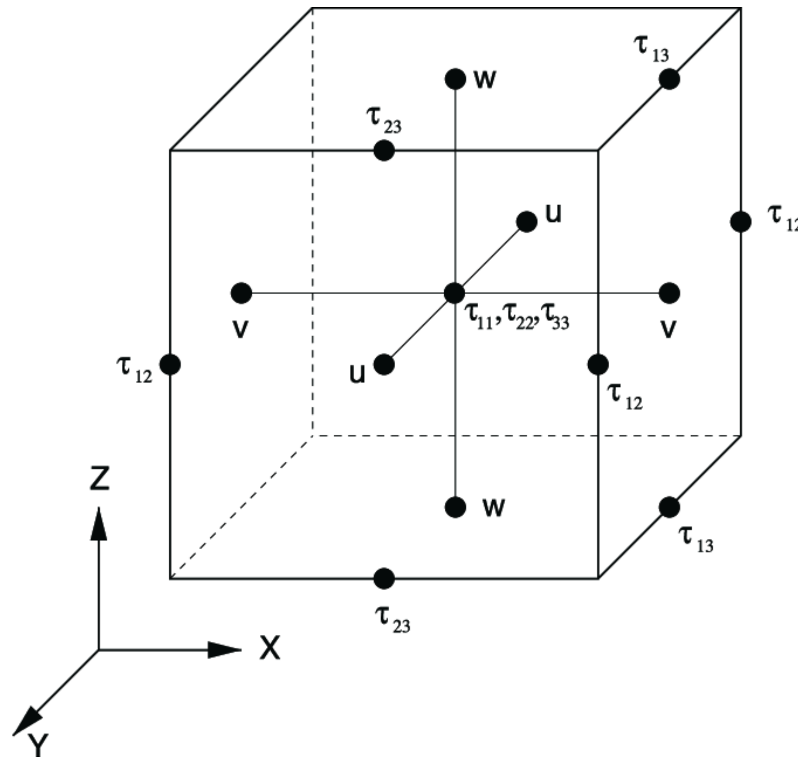
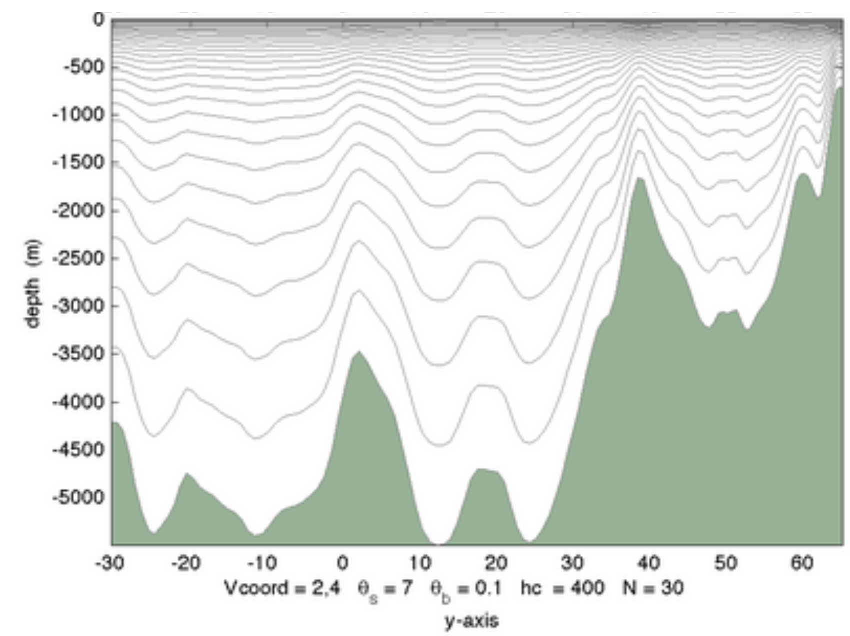
Presentation of advection schemes (moment and tracers) and viscosity/diffusion operators within CROCO

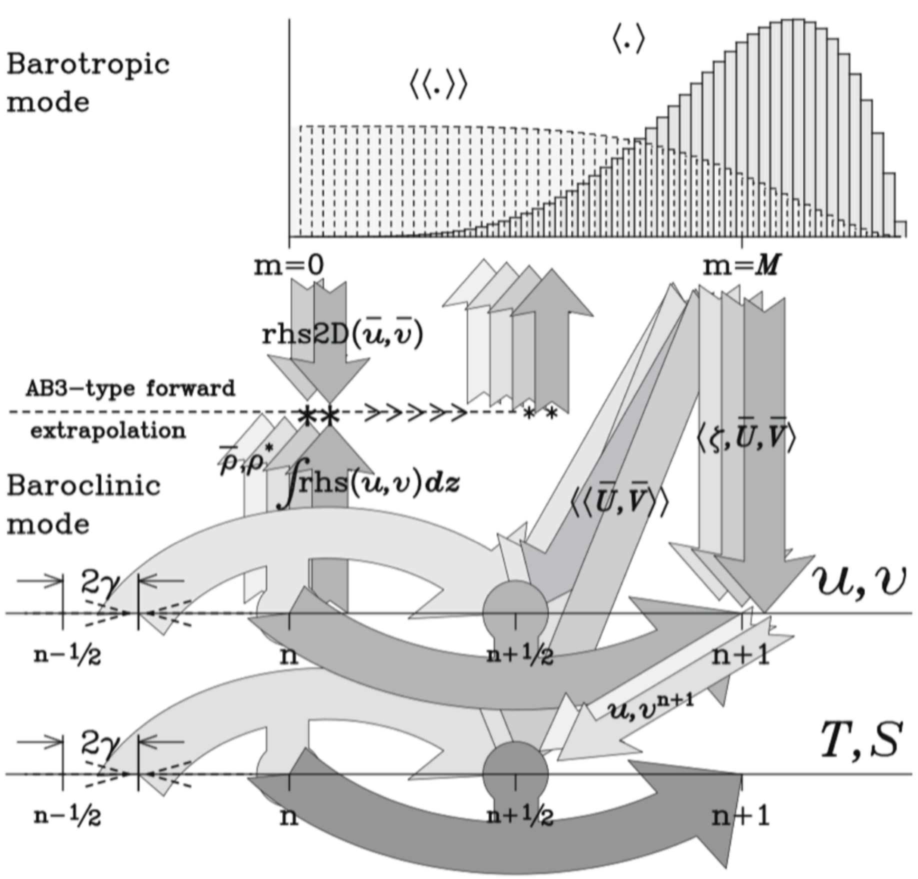
Rachid Benshila, Florian Lemarié

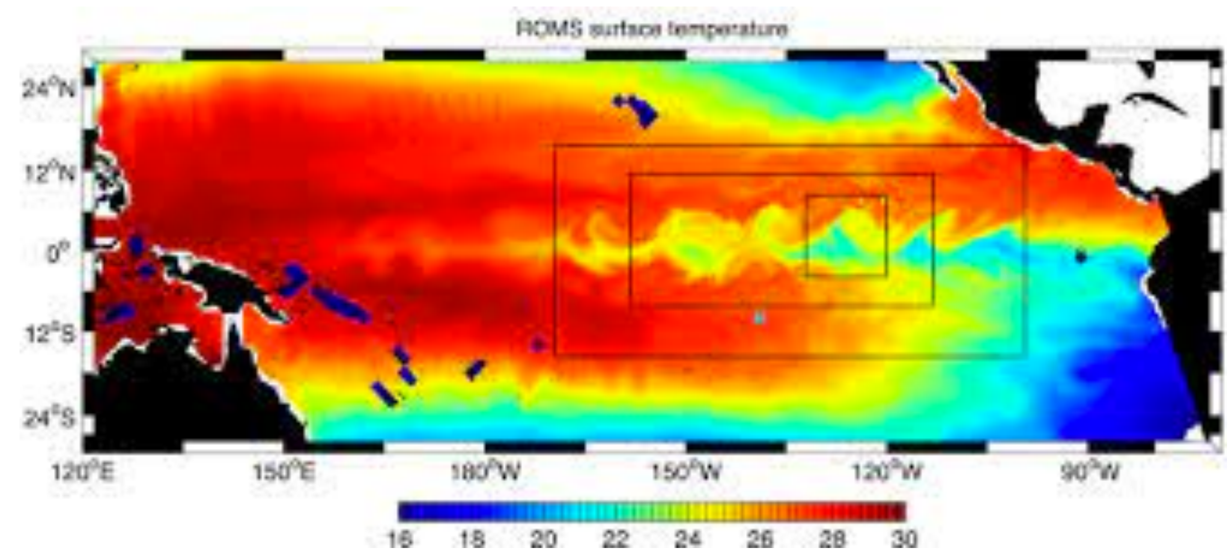
CROCO numerics : basic reminders











Reminder : Primitive equations

$$\frac{\partial u}{\partial t} + \vec{\nabla}.\left(\vec{\mathbf{v}}u\right) - fv = -\frac{\partial \phi}{\partial x} + \mathcal{F}_u + \mathcal{D}_u$$
$$\frac{\partial v}{\partial t} + \vec{\nabla}.\left(\vec{\mathbf{v}}v\right) + fu = -\frac{\partial \phi}{\partial y} + \mathcal{F}_v + \mathcal{D}_v$$

$$\frac{\partial C}{\partial t} + \vec{\nabla}.\left(\vec{\mathbf{v}}C\right) = \mathcal{F}_C + \mathcal{D}_C$$

$$\rho = \rho(T, S, P)$$

$$\frac{\partial \phi}{\partial z} = -\frac{\rho g}{\rho_0}$$

$$\vec{\nabla}.\vec{\mathbf{v}} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Primitive Equations Reynolds averaged

$$X = \langle X \rangle + X'$$

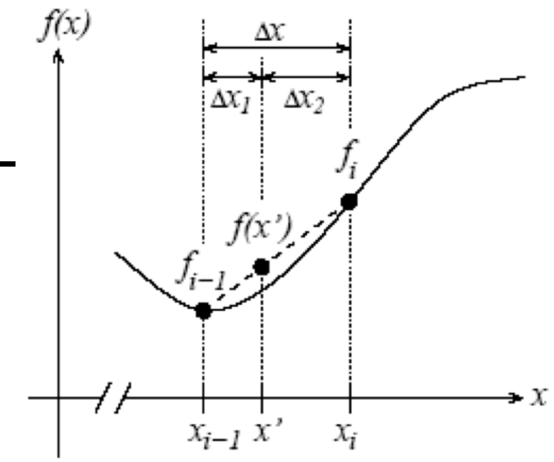
$$\begin{aligned}
\frac{D\langle \mathbf{u}_h \rangle}{Dt} + f\mathbf{k} \times \langle \mathbf{u}_h \rangle &= \frac{\boldsymbol{\nabla}_h p}{\rho_0} - \boldsymbol{\nabla}_h \cdot \langle \mathbf{u}'_h \mathbf{u}'_h \rangle - \partial_z \langle w' \mathbf{u}'_h \rangle \\
\partial_z p &= -g\rho' \\
\boldsymbol{\nabla} \cdot \langle \mathbf{u} \rangle &= 0 \\
\frac{D\langle T \rangle}{Dt} &= -\frac{\partial_z Q_s}{\rho_0 C_{p,o}} - \boldsymbol{\nabla}_h \cdot \langle \mathbf{u}'_h T' \rangle - \partial_z \langle w' T' \rangle \\
\frac{D\langle S \rangle}{Dt} &= -\boldsymbol{\nabla}_h \cdot \langle \mathbf{u}'_h S' \rangle - \partial_z \langle w' S' \rangle \\
\rho &= \rho_{\mathrm{eos}}(\langle T \rangle, \langle S \rangle, z)
\end{aligned}$$

c

$$\frac{D\langle X \rangle}{Dt} = \partial_t \langle X \rangle + \boldsymbol{\nabla} \cdot \langle X \rangle \langle \mathbf{u} \rangle$$

Advection schemes

Reminders: Order of a scheme



$$\frac{\partial q}{\partial t} + C_0 \frac{\partial q}{\partial x} = 0$$

Goal: to find an approximation of partial derivatives
=> taylor series at point x

$$f(x \pm \epsilon) = f(x) + \sum_{j=1}^{\infty} \epsilon^j \frac{(\pm 1)^j}{j!} \frac{\partial^j}{\partial x^j} f$$

$$q_i' = \frac{q_{i+1} - q_i}{\Delta x} + E \quad ; \quad E = -q_i'' \left(\frac{\Delta x}{2!}\right) + \mathcal{O}(\Delta x^2)$$ **Order 1**

$$q_i' = \frac{q_i - q_{i-1}}{\Delta x} + E \quad ; \quad E = q_i'' \left(\frac{\Delta x}{2!}\right) + \mathcal{O}(\Delta x^2)$$ **Order 1**

$$q_i' = \frac{q_{i+1} - q_{i-1}}{2\Delta x} + E \quad ; \quad E = q_i''' \left(\frac{\Delta x^2}{3!2}\right) + \mathcal{O}(\Delta x^3)$$ **Order 2**

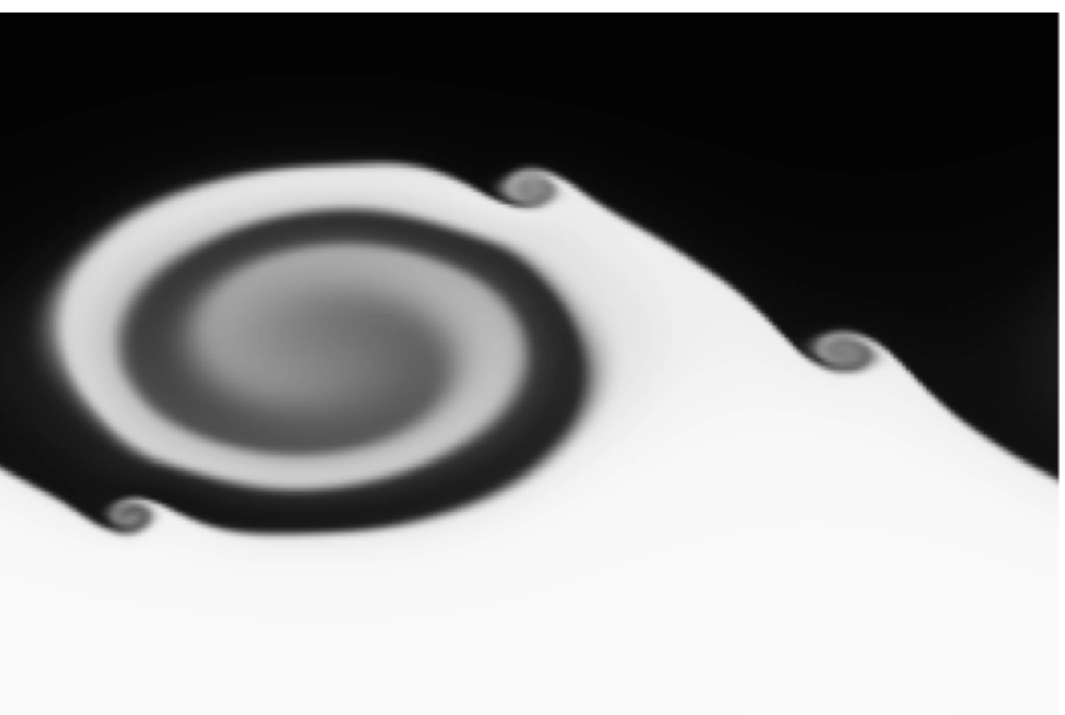
Order 3

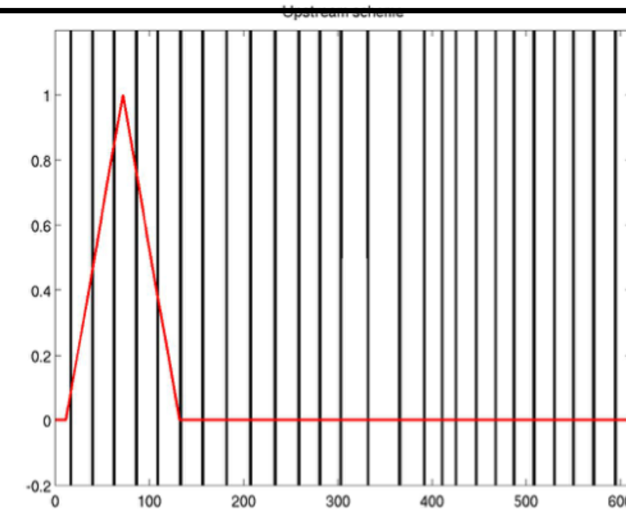
$$q_i' = \frac{4}{3}\frac{q_{i+1} - q_{i-1}}{2\Delta x} - \frac{1}{3}\frac{q_{i+2} - q_{i-2}}{4\Delta x} + \frac{q_{i+1} - 3q_i + 3q_{i-1}}{12\Delta x} + E \quad ; \quad E = q_i'''' \left(\frac{\Delta x^3}{4!}\right) + \mathcal{O}(\Delta x^4)$$

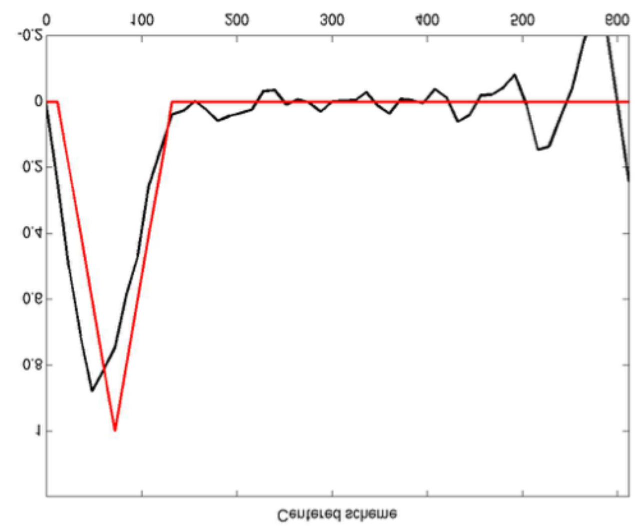
$$q_i' = \frac{4}{3}\frac{q_{i+1} - q_{i-1}}{2\Delta x} - \frac{1}{3}\frac{q_{i+2} - q_{i-2}}{4\Delta x} + E \quad ; \quad E = q_i''''' \left(\frac{\Delta x^4}{5!}\right) + \mathcal{O}(\Delta x^5)$$ **Order 4**

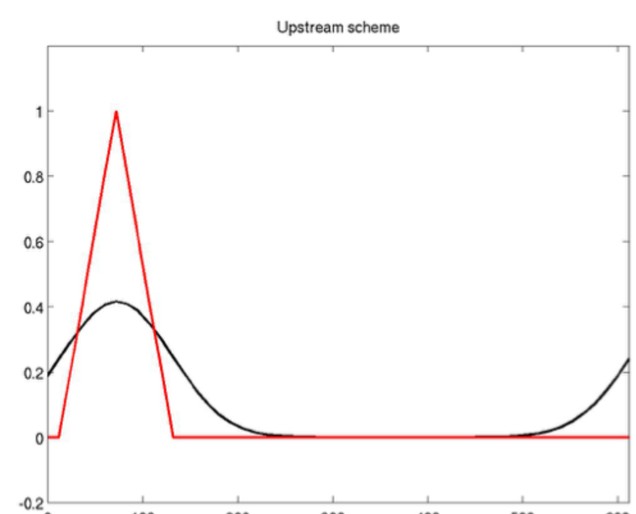
A "good" scheme

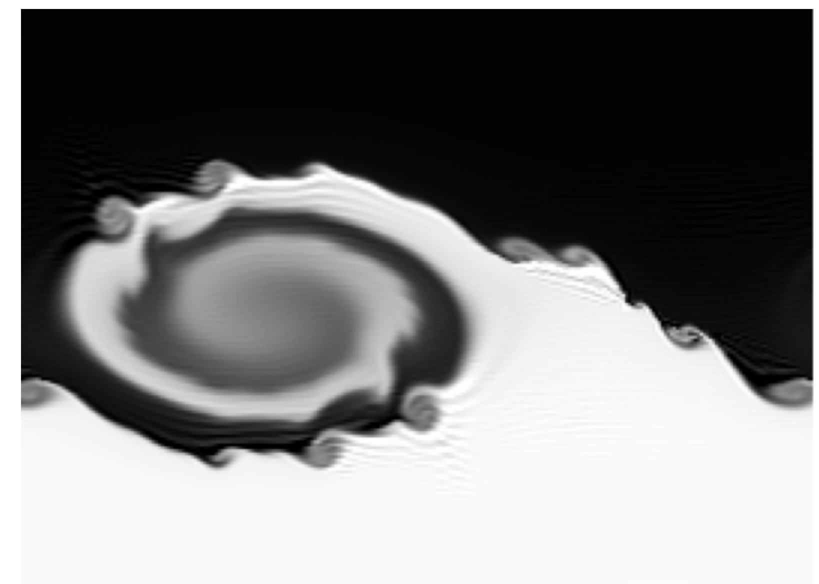
- Stable, CFL= $\Delta t < \dfrac{\Delta x}{V_{max}}$
- Precise
- Low dispersive
- Not too diffusive
- Cheap

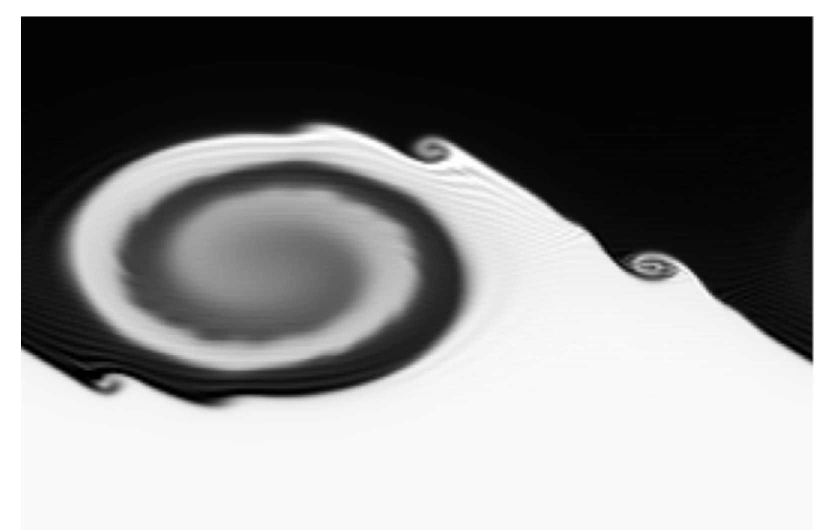








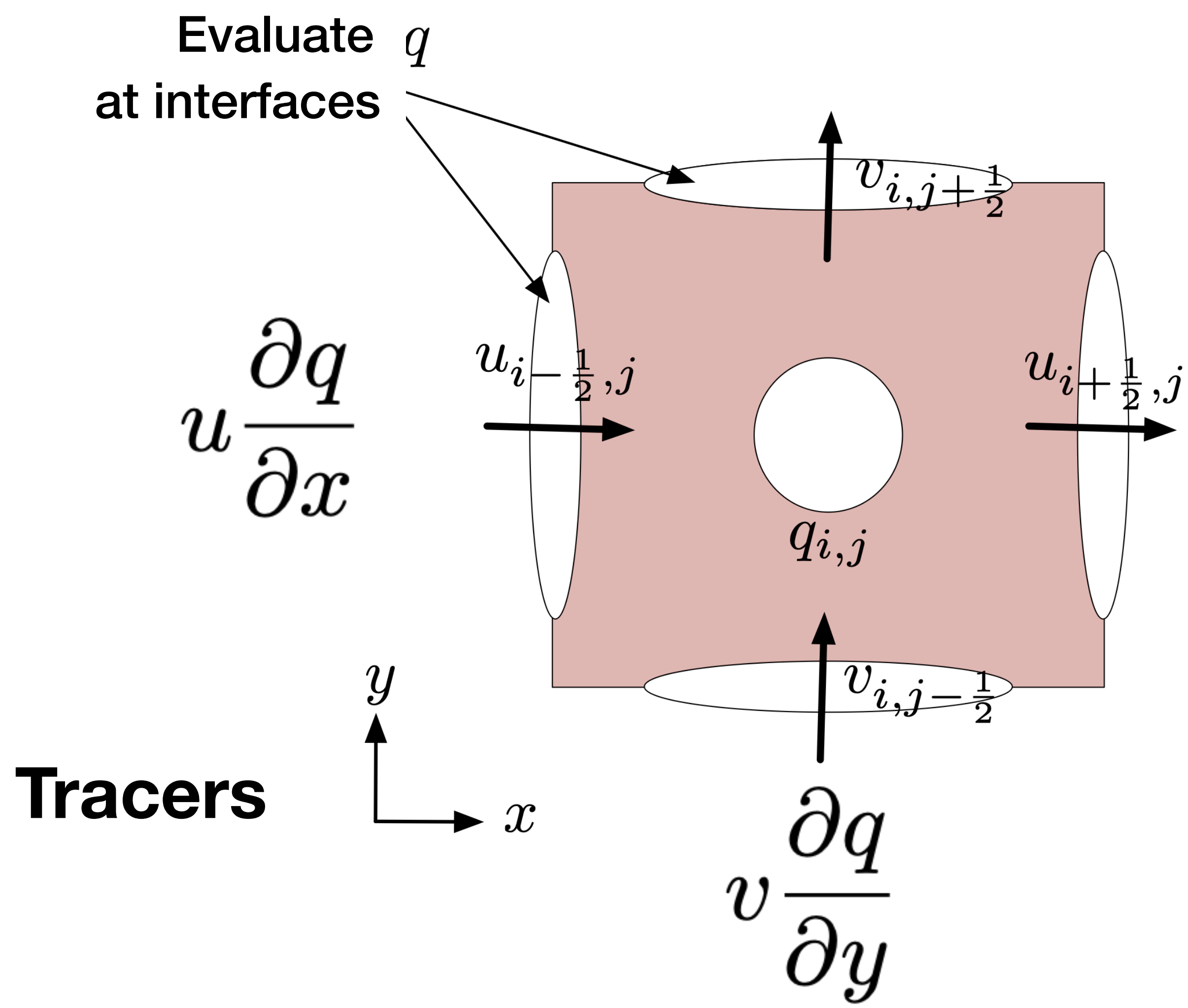


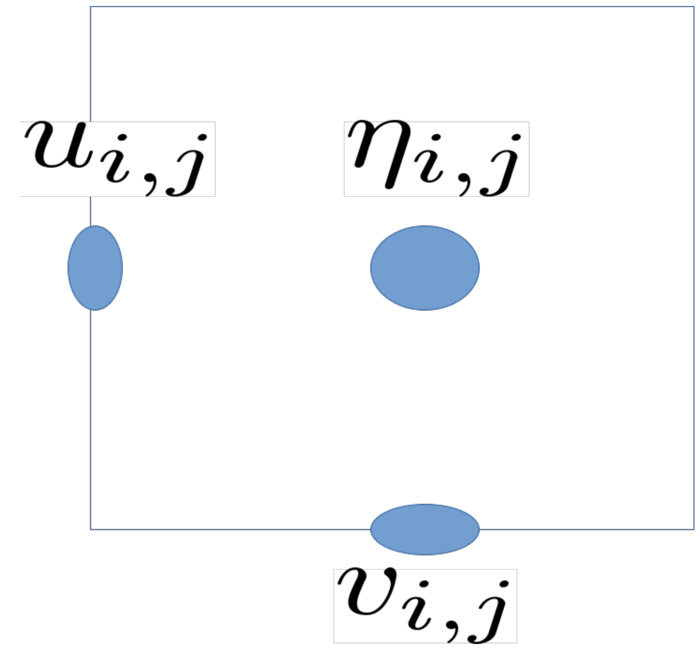


Options for advection

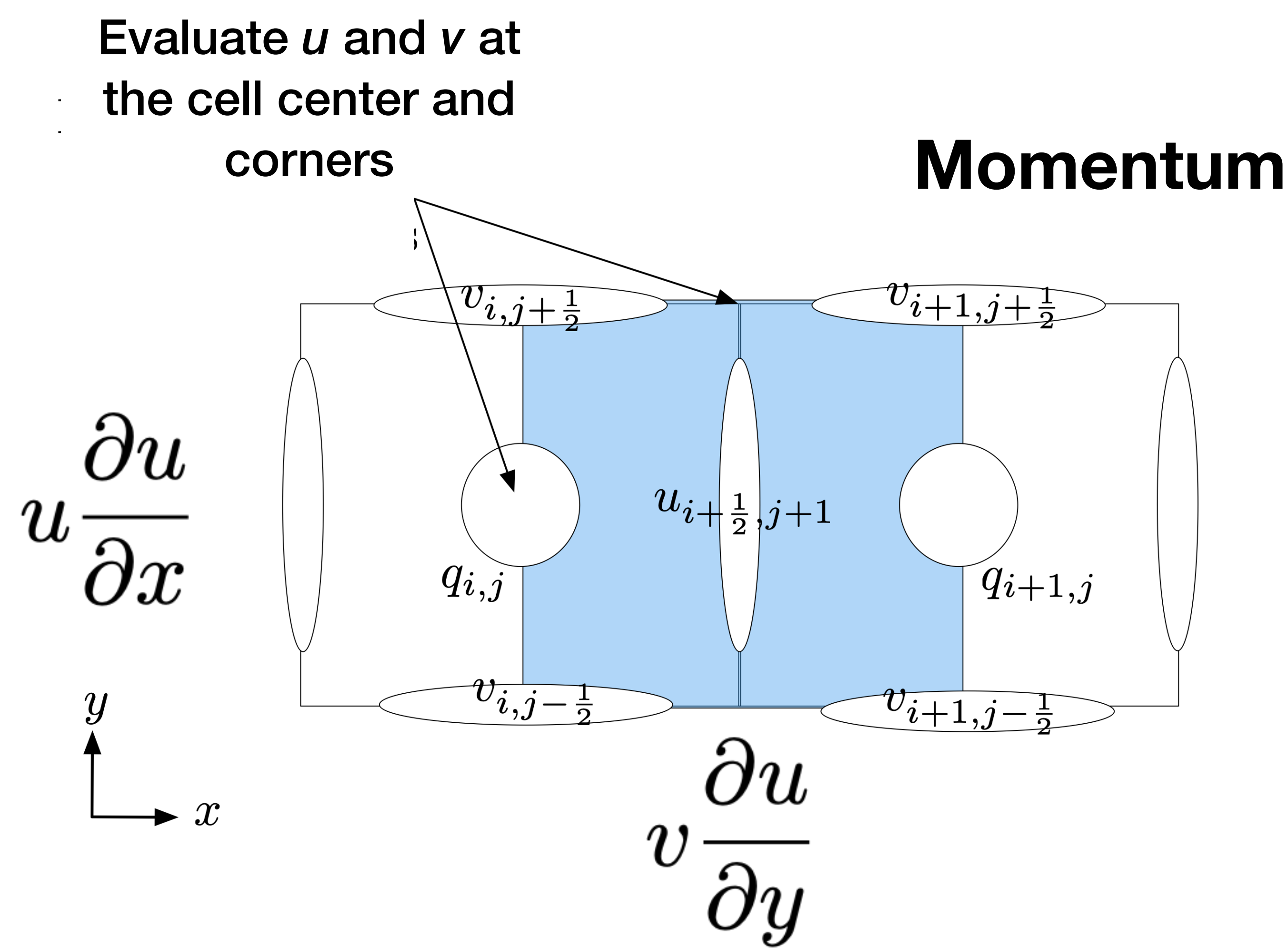
Equation	horizontal	vertical
Momentum 3D (UV_ADV) (+ eq W_ en NBQ)	UV_HADV_TVD UV_HADV_C2 UV_HADV_UP3 UV_HADV_C4 UV_HADV_UP5 UV_HADV_C6 UV_HADV_WENO5	UV_VADV_TVD UV_VADV_C2 UV_VADV_SPLINES UV_VADV_WENO5
Tracers	TS_HADV_UP3 TS_HADV_RSUP3 TS_HADV_C4 TS_HADV_WENO5 TS_HADV_UP5 TS_HADV_C6 TS_HADV_RSUP5	TS_VADV_TVD TS_VADV_C2 TS_VADV_SPLINES TS_VADV_AKIMA TS_VADV_WENO5
Momentum 2D	M2_HADV_UP3 M2_HADV_C2	

The Advection Problem: An Interpolation Problem



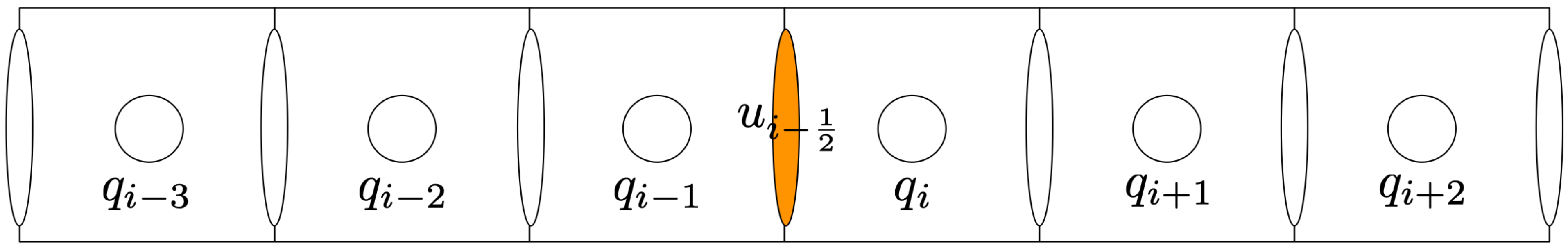


Arakawa C-grid



Horizontal advection

HADV_C2, HADV_UP3, HADV_C4,HADV_UP5, HADV_C6



$$\partial_x(uq)\big|_{x=x_i} = \frac{1}{\Delta x_i}\left\{u_{i+\frac{1}{2}}\widetilde{q}_{i+\frac{1}{2}} - u_{i-\frac{1}{2}}\widetilde{q}_{i-\frac{1}{2}}\right\}$$

$$\widetilde{q}^{\,\mathrm{C2}}_{i-\frac{1}{2}} = \frac{q_i + q_{i-1}}{2}$$

$$\widetilde{q}^{\,\mathrm{C4}}_{i-\frac{1}{2}} = (\frac{7}{6})\widetilde{q}^{\,\mathrm{C2}}_{i-\frac{1}{2}} - (\frac{1}{12})(q_{i+1} + q_{i-2})$$

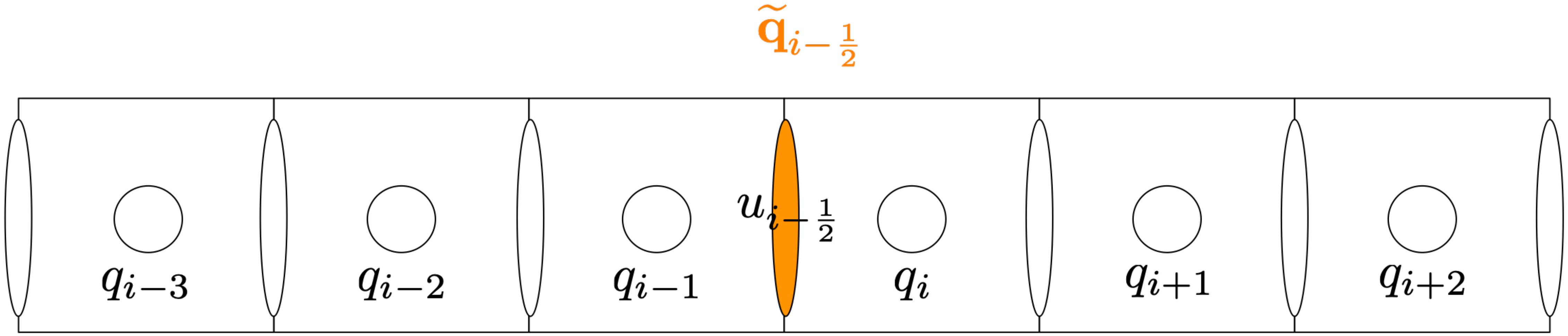
$$\widetilde{q}^{\,\mathrm{UP3}}_{i-\frac{1}{2}} = \widetilde{q}^{\,\mathrm{C4}}_{i-\frac{1}{2}} + \mathrm{sign}(\frac{1}{12}, u_{i-\frac{1}{2}})(q_{i+1} - 3q_i + 3q_{i-1} - q_{i-2})$$

$$\widetilde{q}^{\,\mathrm{C6}}_{i-\frac{1}{2}} = (\frac{8}{5})\widetilde{q}^{\,\mathrm{C4}}_{i-\frac{1}{2}} - (\frac{19}{60})\widetilde{q}^{\,\mathrm{C2}}_{i-\frac{1}{2}} + (\frac{1}{60})(q_{i+2} + q_{i-3})$$

$$\widetilde{q}^{\,\mathrm{UP5}}_{i-\frac{1}{2}} = \widetilde{q}^{\,\mathrm{C6}}_{i-\frac{1}{2}} - \mathrm{sign}(\frac{1}{60}, u_{i-\frac{1}{2}})(q_{i+2} - 5q_{i+1} + 10q_i - 10q_{i-1} + 5q_{i-2} - q_{i-3})$$

Horizontal advection

HADV_RSUP3,HADV_RSUP5



$$\partial_x(uq)|_{x=x_i} = \frac{1}{\Delta x_i}\left\{u_{i+\frac{1}{2}}\widetilde{q}_{i+\frac{1}{2}} - u_{i-\frac{1}{2}}\widetilde{q}_{i-\frac{1}{2}}\right\}$$

$$\widetilde{q}^{\,\mathrm{UP3}}_{i-\frac{1}{2}} = \widetilde{q}^{\,\mathrm{C4}}_{i-\frac{1}{2}} + \mathrm{sign}\left(\frac{1}{12}, u_{i-\frac{1}{2}}\right)(q_{i+1} - 3q_i + 3q_{i-1} - q_{i-2})$$

$$\widetilde{q}^{\,\mathrm{UP5}}_{i-\frac{1}{2}} = \widetilde{q}^{\,\mathrm{C6}}_{i-\frac{1}{2}} - \mathrm{sign}\left(\frac{1}{60}, u_{i-\frac{1}{2}}\right)(q_{i+2} - 5q_{i+1} + 10q_i - 10q_{i-1} + 5q_{i-2} - q_{i-3})$$

=> Schemes Split-UP3 et Split-UP5

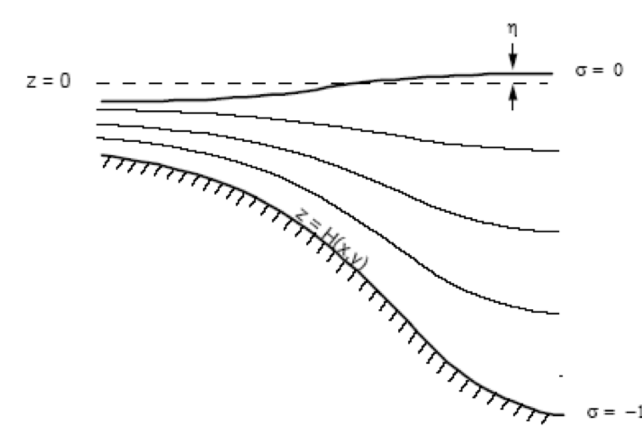
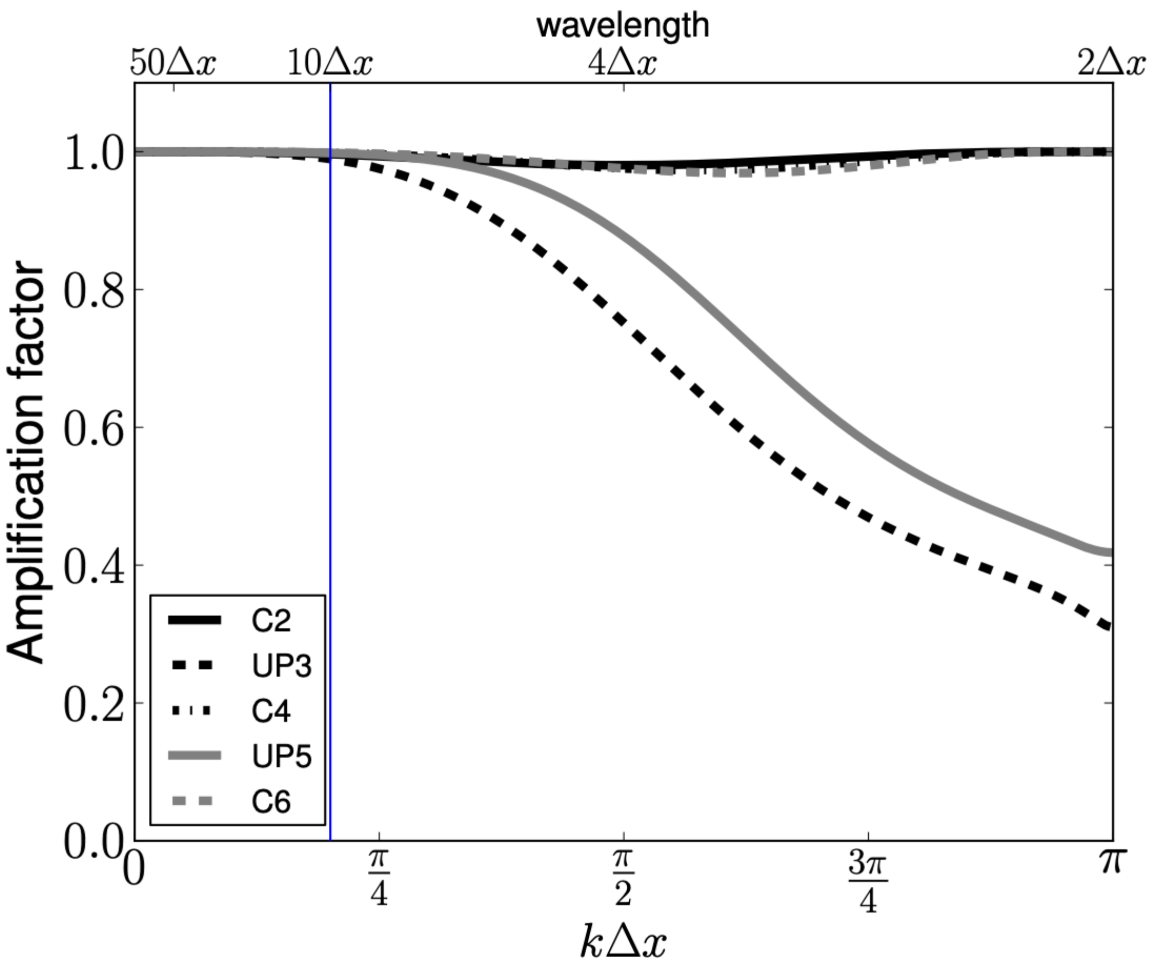
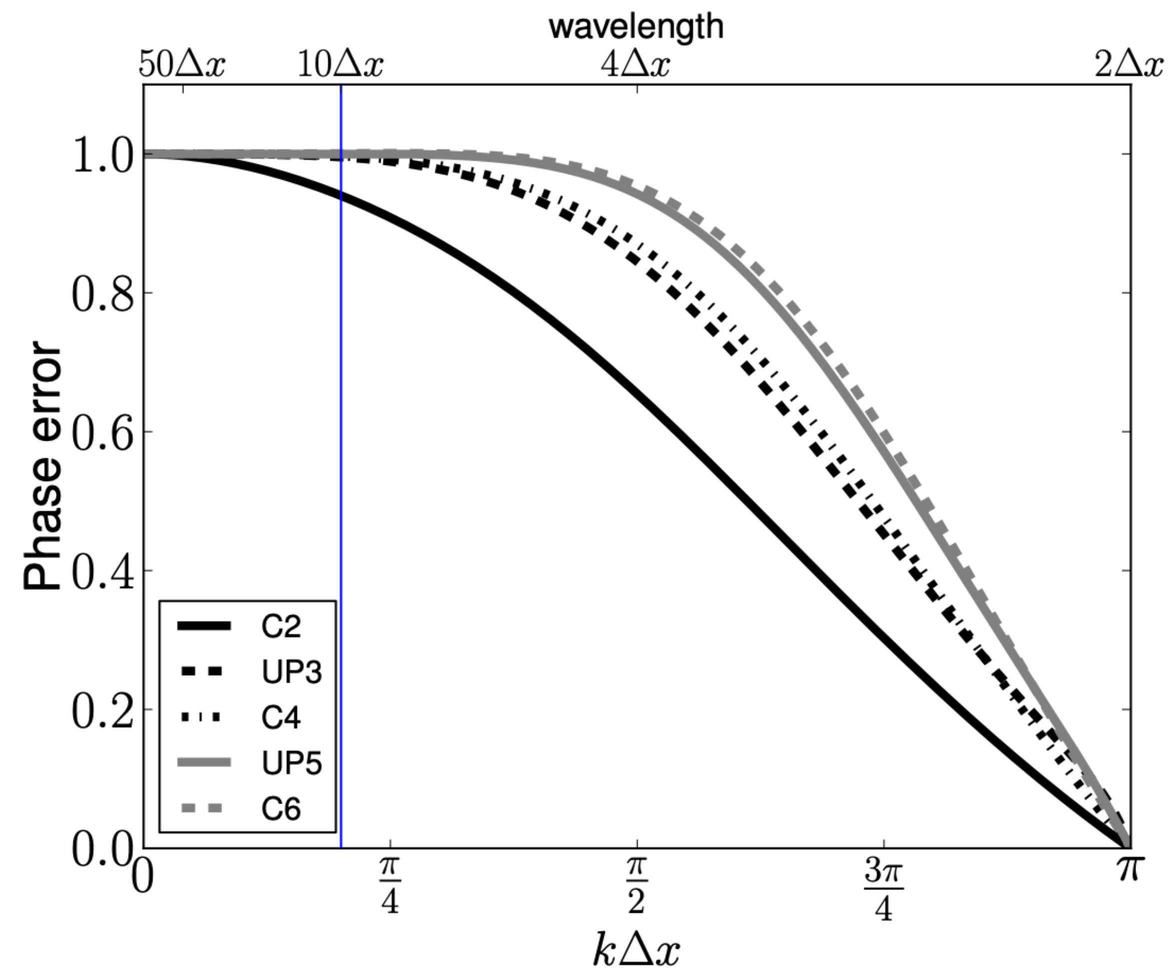


Figure 1. The sigma coordinate system.

Properties of linear schemes





$$q = q_0 e^{ikx-\omega t}$$

$$q^n = (A)^n q_0, we\ want\ A <= 1$$

Note : finite differences vs. finite volumes

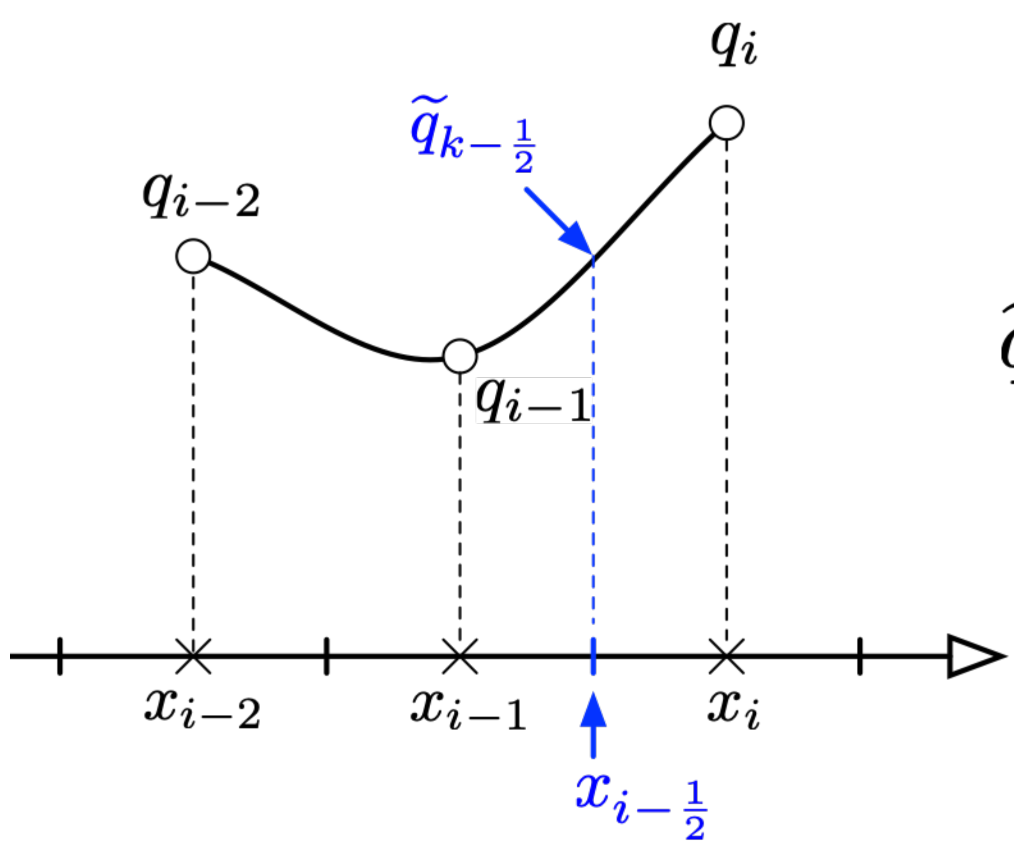
Classical UP3 scheme

$$\frac{q(x_i+\Delta x)-q(x_i-\Delta x)}{2\Delta x} = \partial_x q(x_i) + \frac{1}{6}\Delta x^2 \partial_x^3 q(x_i) + \mathcal{O}(\Delta x^4)$$

To cancel order 2 error

$$\widetilde{q}^{\mathrm{UP3}}_{i-\frac{1}{2}} = \frac{q_i+q_{i-1}}{2} - \frac{1}{6}(q_i - 2q_{i-1} + q_{i-2})$$

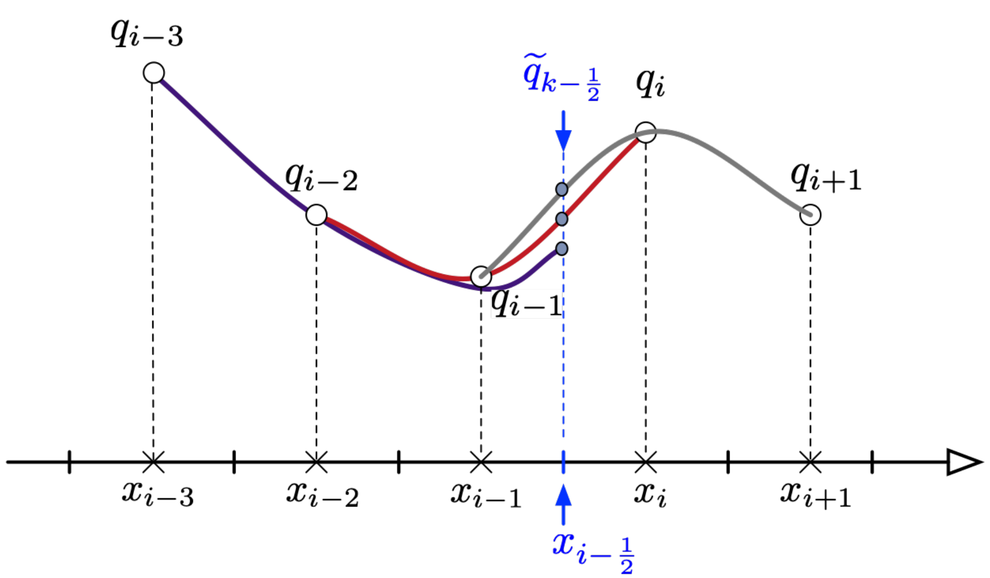
UP3 scheme in the sense of finite volumes (ex Leonard 95)



$$\widetilde{q}_{i-\frac{1}{2}} = \frac{q_i+q_{i-1}}{2} - \frac{1}{8}(q_i - 2q_{i-1} + q_{i-2})$$

WENO schemes (e.g Acker et al, 2016), *TS_HADV_WENO5*

Non-linear weighting between 3 interface value evaluations



$$\widetilde{q}_{k-\frac{1}{2}} = w_0 \widetilde{q}^{(0)}_{k-\frac{1}{2}} + w_1 \widetilde{q}^{(1)}_{k-\frac{1}{2}} + w_2 \widetilde{q}^{(2)}_{k-\frac{1}{2}}$$

1. Convexity

ENO property (essentially non-oscillatory)

5th order if q(x) is smooth

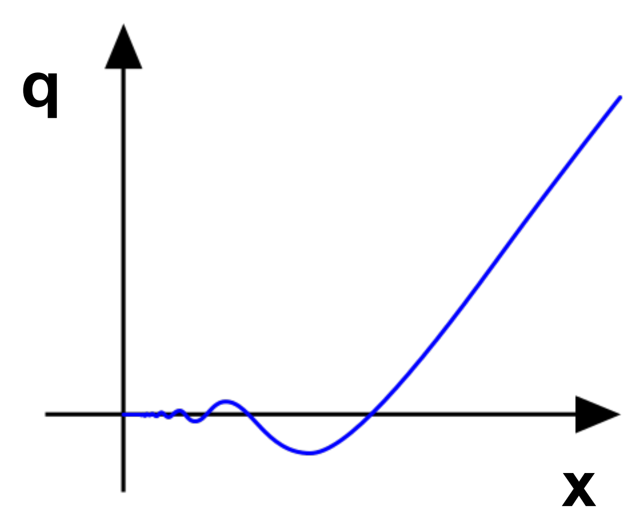
$$\forall N, \exists M, \sum_{j=1}^{N-1} |q_{j+1} - q_j| \leq M$$

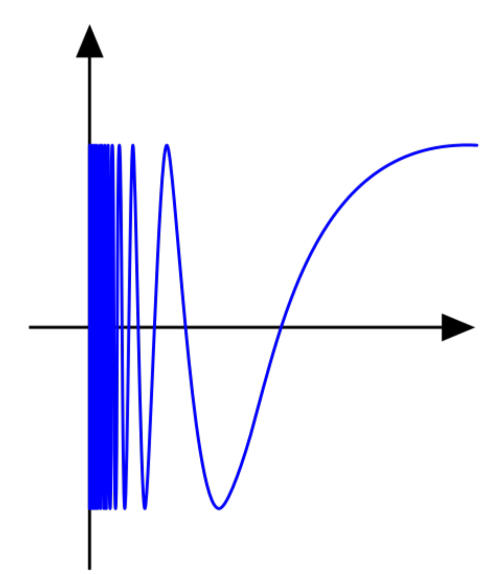
1. ENO property (it satisfies a Total Variation Bounded constraint)

But does not preserve the monotony !!!!

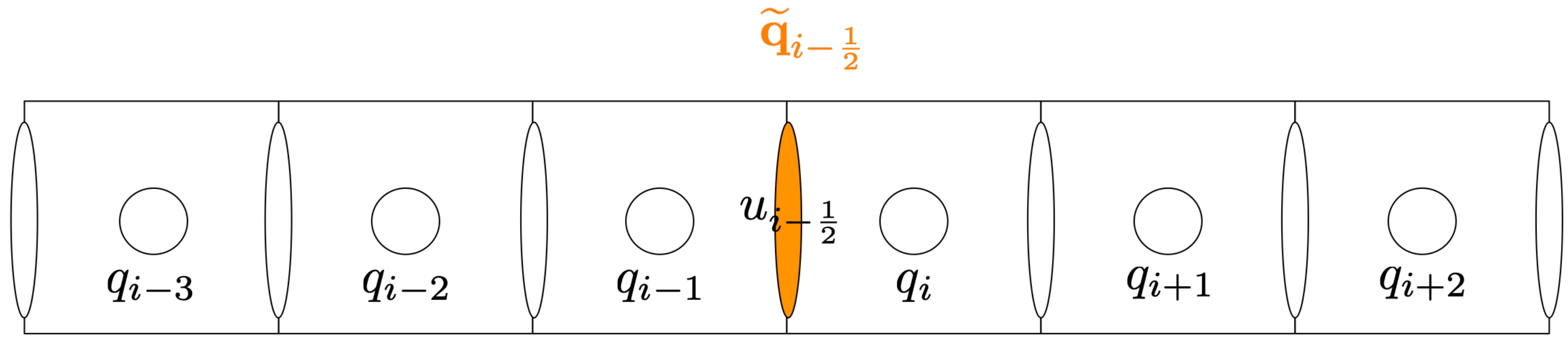
=> nevertheless: preferred choice for biogeochmic tracers

BIO_HADV_WENO5





TVD schemes, *UV_HADV_TVD*



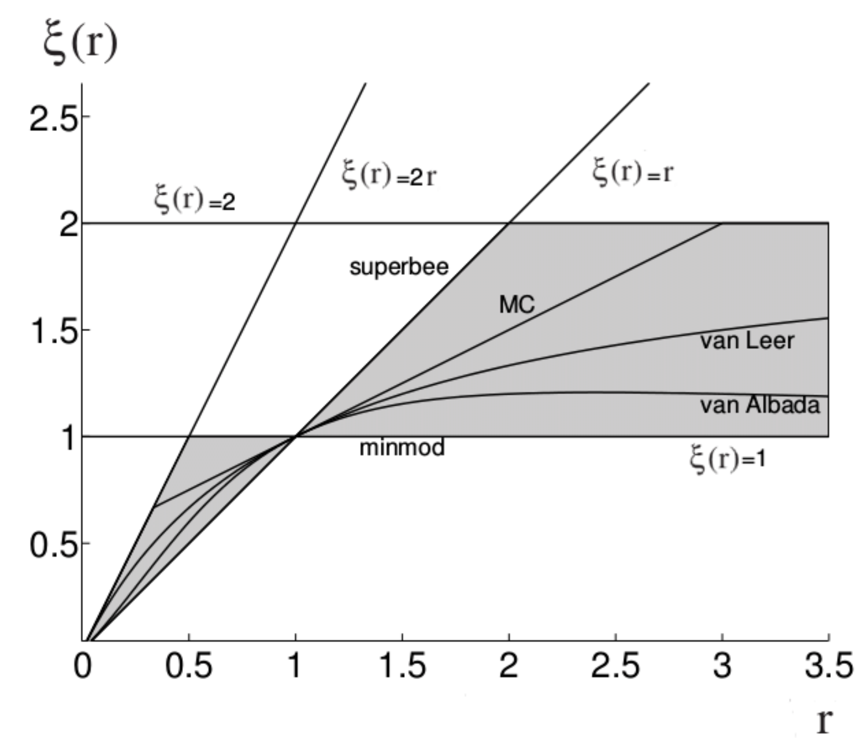
Ponderation between upwind et C2

$$\widetilde{q}^{TVD}_{i-\frac{1}{2}} = (1-\xi)\widetilde{q}^{C2}_{i-\frac{1}{2}} + \xi\widetilde{q}^{UPW}_{i-\frac{1}{2}}$$

$$u_{i-\frac{1}{2}} > 0 \quad : \quad \widetilde{q}^{TVD}_{i-\frac{1}{2}} = q_{i-1} \quad ; \quad r = \frac{q_i - q_{i-1}}{q_{i+1} - q_i}$$

$$u_{i-\frac{1}{2}} < 0 \quad : \quad \widetilde{q}^{TVD}_{i-\frac{1}{2}} = q_i \quad ; \quad r = \frac{q_{i+2} - q_{i+1}}{q_{i+1} - q_i}$$

$$\xi = f(r)$$



1. Total Variation Diminishing => monotonic
2. But order 1 !!!

$$\forall N, \quad \left(\sum_{j=1}^{N-1} |q_{j+1} - q_j|\right)^{n+1} \leq \left(\sum_{j=1}^{N-1} |q_{j+1} - q_j|\right)^{n}$$

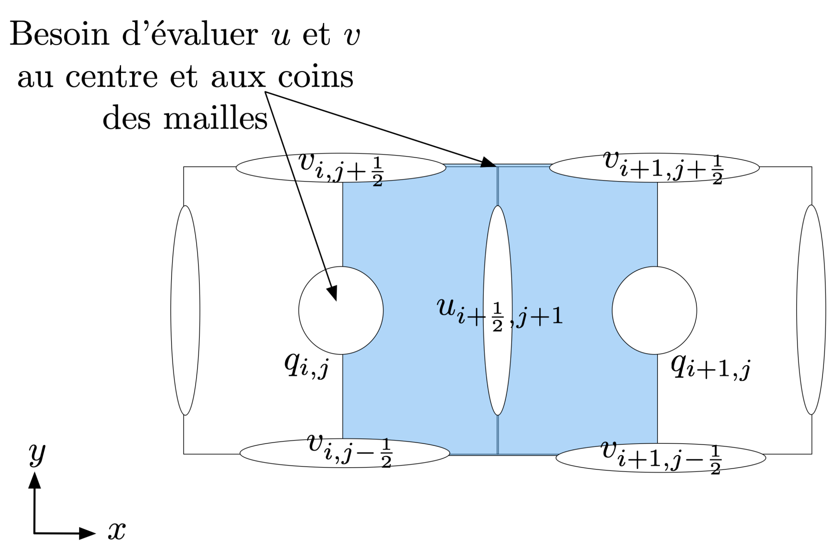
Non-linear terms (rhs3d.F)

Lilly (1965) type formulation

$$\partial_t(\mathrm{Hz}u) + \partial_x\left((\mathrm{Hz}\ u)u\right) + \partial_y\left((\mathrm{Hz}\ v)\ u\right) + \ldots$$
$$\partial_t(\mathrm{Hz}v) + \partial_x\left((\mathrm{Hz}\ u)v\right) + \partial_y\left((\mathrm{Hz}\ v)\ v\right) + \ldots$$

UV_HADV_UP3

$$\tilde{q}^{UP3}_{i-\frac{1}{2}} = \tilde{q}^{C4}_{i-\frac{1}{2}} + \mathrm{sign}(\frac{1}{12}, u_{i-\frac{1}{2}})(q_{i+1} - 3q_i + 3q_{i-1} - q_{i-2})$$



$$\left((\widetilde{\mathrm{Hz}\ u})u\right)_{i,j} = (\widetilde{\mathrm{Hz}\ u})^{\mathrm{C4}}_{i,j}\ \tilde{u}^{\mathrm{UP3}}_{i,j}$$

$$\left((\widetilde{\mathrm{Hz}\ v})u\right)_{i+\frac{1}{2},j+\frac{1}{2}} = (\widetilde{\mathrm{Hz}\ v})^{\mathrm{C4}}_{i+\frac{1}{2},j+\frac{1}{2}}\ \tilde{u}^{\mathrm{UP3}}_{i+\frac{1}{2},j+\frac{1}{2}}$$

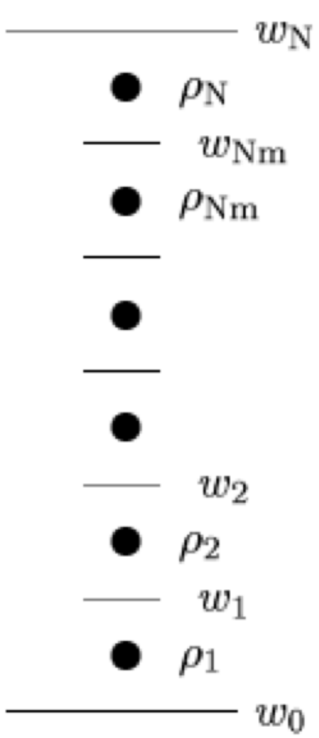
with the choice of the upwind direction made according to:

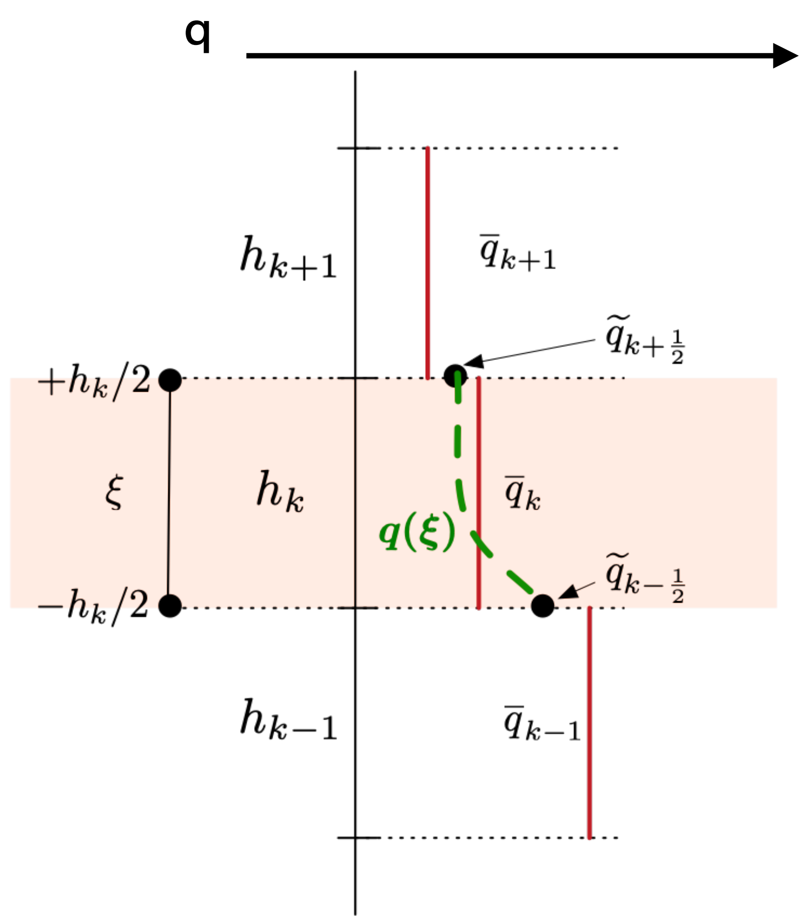
$$u^{\mathrm{upw}}_{i,j} = u_{i+\frac{1}{2},j} + u_{i-\frac{1}{2},j}, \qquad v^{\mathrm{upw}}_{i+\frac{1}{2},j+\frac{1}{2}} = (\mathrm{Hz}\ v)_{i,j+\frac{1}{2}} + (\mathrm{Hz}\ v)_{i+1,j+\frac{1}{2}}$$

Vertical advection schemes : *VADV_SPLINES*

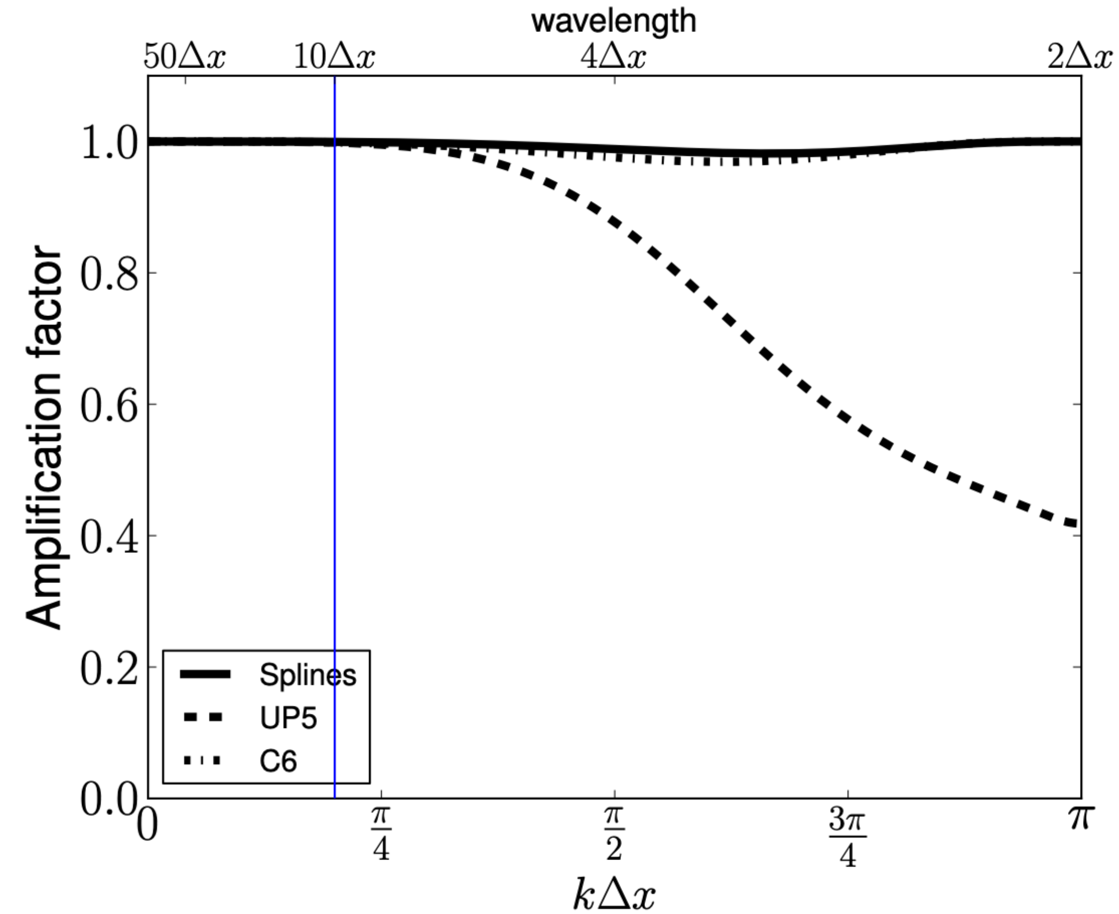
Fluxes obtained as a solution of the tri-diagonal problem:

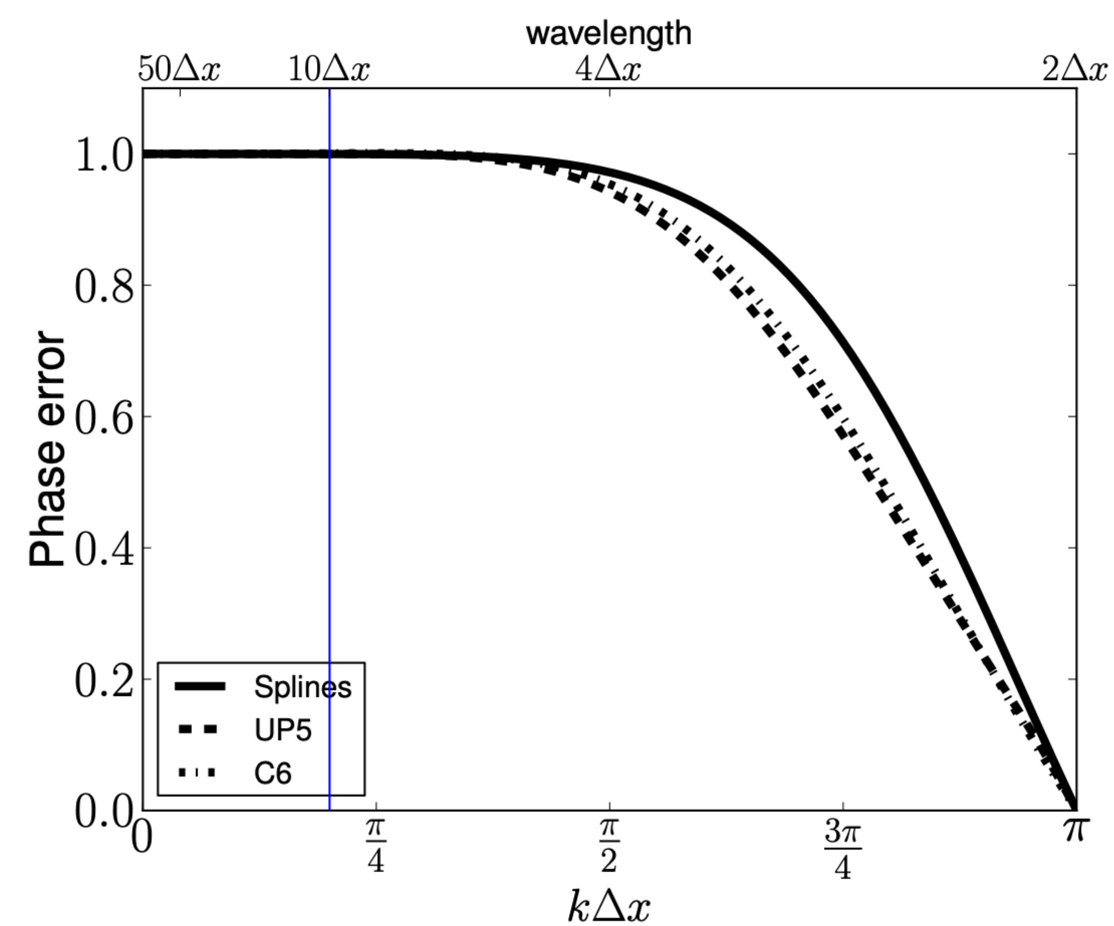
$$\mathrm{Hz}_{k+1}\widetilde{q}_{k-\frac{1}{2}} + 2(\mathrm{Hz}_k + \mathrm{Hz}_{k+1})\widetilde{q}_{k+\frac{1}{2}} + \mathrm{Hz}_k\widetilde{q}_{k+\frac{3}{2}} = 3(\mathrm{Hz}_k\overline{q}_{k+1} + \mathrm{Hz}_{k+1}\overline{q}_k)$$





$$q(\xi) = \overline{q}_k + \frac{\widetilde{q}_{k+\frac{1}{2}} - \widetilde{q}_{k-\frac{1}{2}}}{h_k}\xi + 6\left(\frac{\widetilde{q}_{k+\frac{1}{2}} + \widetilde{q}_{k-\frac{1}{2}}}{2} - \overline{q}_k\right)\left(\frac{\xi^2}{h_k^2} - \frac{1}{12}\right)$$





Semi-implicit formulation

V_ADV_IMP

Idea: split vertical velocity into an explicit contribution and an implicit contribution (Shchepetkin 2015)

$$\Omega = \mathbf{\Omega}^{(\mathrm{e})} + \mathbf{\Omega}^{(\mathrm{i})}, \qquad \mathbf{\Omega}^{(\mathrm{e})} = \frac{\Omega}{f(\alpha^z_{\mathrm{adv}}, \alpha_{\mathrm{max}})}, \quad f(\alpha^z_{\mathrm{adv}}, \alpha_{\mathrm{max}}) = \begin{cases} 1, & \alpha^z_{\mathrm{adv}} \leq \alpha_{\mathrm{max}} \\ \alpha/\alpha_{\mathrm{max}}, & \alpha^z_{\mathrm{adv}} > \alpha_{\mathrm{max}} \end{cases}$$

$\rightarrow$ $\mathbf{\Omega}^{(\mathrm{e})}$ **integrated with an explicit scheme (max CFL)**

$\rightarrow$ $\mathbf{\Omega}^{(\mathrm{i})}$ **integrated with an Euler uwind implicit scheme** icite

Configuration	résolution	ancien dt	nouveau dt
BENGUELA (Penven et al)	25 km	6300s	7140s
OMAN (Vic et al)	2 km	160s	470s

No control over errors
Obligation to use a linear scheme on the explicit part

Scheme C4 and « akima » (Van leer 1977)

V_ADV_C4, VADV_AKIMA

$$\widetilde{q}^{\,C4}_{k-\frac{1}{2}} = (\frac{7}{6})\widetilde{q}^{\,C2}_{k-\frac{1}{2}} - (\frac{1}{12})(q_{k+1} + q_{k-2})$$

$$= \widetilde{q}^{\,C2}_{k-\frac{1}{2}} - \frac{1}{6}\left(d_k - d_{k-1}\right), \qquad d_k = \frac{\Delta q_{k+\frac{1}{2}} + \Delta q_{k-\frac{1}{2}}}{2}$$

AKIMA : C4 with harmonic mean of the slopes instead of algebric

$$d_k = \begin{cases} \dfrac{2}{\frac{1}{\Delta q_{k+\frac{1}{2}}} + \frac{1}{\Delta q_{k-\frac{1}{2}}}} & \text{si } \Delta q_{k+\frac{1}{2}} \Delta q_{k-\frac{1}{2}} > 0 \\ 0 & \text{sinon} \end{cases}$$

How to choose ???

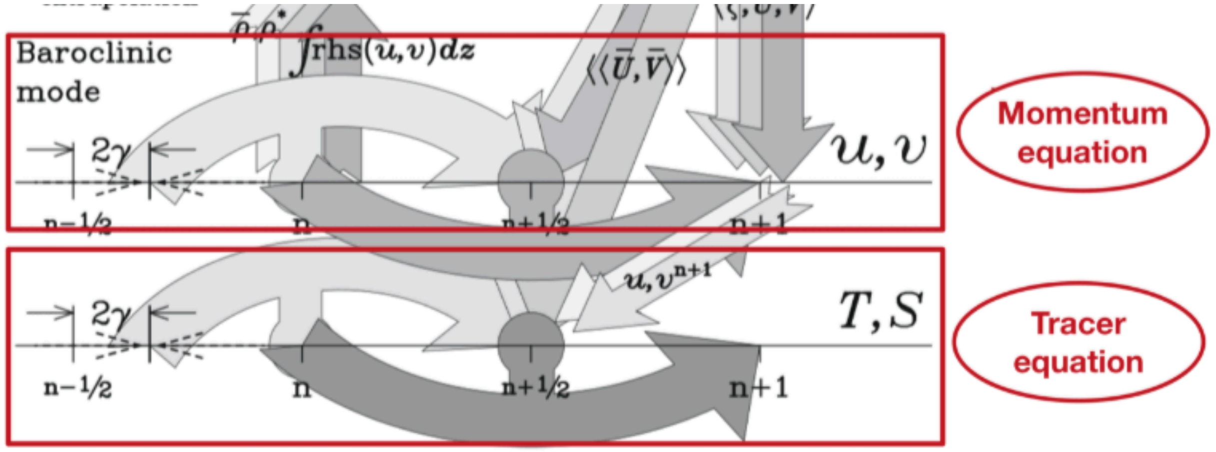
Equation	horizontal	vertical
Moment 3D (UV_ADV) (+ eq W_ en NBQ)	UV_HADV_TVD UV_HADV_C2 UV_HADV_UP3 UV_HADV_C4 UV_HADV_UP5 UV_HADV_C6 UV_HADV_WENO5	UV_VADV_TVD UV_VADV_C2 UV_VADV_SPLINES UV_VADV_WENO5
Traceurs	TS_HADV_UP3 TS_HADV_RSUP3 TS_HADV_C4 TS_HADV_WENO5 TS_HADV_UP5 TS_HADV_C6 TS_HADV_RSUP5	TS_VADV_TVD TS_VADV_C2 TS_VADV_SPLINES TS_VADV_AKIMA TS_VADV_WENO5
Moment 2D	M2_HADV_UP3 M2_HADV_C2	

3920 combinations …

-> choose not to choose
robust default choices
->other error causes
-> physics :
positivity, scales
-> cost (compromise)

+V_VADV_IMP
+BIO_HADV_WENO
+ …

In practice



2 calls per time
-> re-use (or not) the same routines

Moment : 2 calls to *rhs3d.F*
with also:

- *u_hadv_order5.h*
- *uvhadv_tvd.h*
- *uhdiff_order5.h*

t(nx,ny,nz,3)

But also :

Different schemes at predictor (cheaper) and at corrector steps
ex : C6+WENO5, C4+UP3

Tracers : *pre_step3d.F, step3d_t.F*
with also :
- *t3dadv_order5.h*
- *compute_vert_tracer_fluxes.h*

define PREDICTOR (traceurs)

nnew /= 3 (momentum)

u(nx,ny,nz,3)

Viscous operators and diffusion

Viscous operator shape (Wajsowic, 1993)

$$< u'u' + u'v' >$$

UV_VIS2 Viscous tensor (*uv3dmix.F*)

$$< v'v' + u'v' >$$

$$\boldsymbol{\sigma}(\mathbf{u}_h) = \begin{pmatrix} \partial_x u - \partial_y v & \partial_y u + \partial_x v \\ \partial_x v + \partial_y u & -(\partial_x u - \partial_y v) \end{pmatrix} \qquad \mathbf{u}_h = [u, v]$$

the viscosity operator is given by

$$-\boldsymbol{\nabla}_h \cdot \langle \mathbf{u}'_h \mathbf{u}'_h \rangle = \frac{1}{\mathrm{Hz}} \boldsymbol{\nabla}_h \cdot (A_M \ \mathrm{Hz} \ \boldsymbol{\sigma}) \,, \qquad A_M \leftrightarrow \mathrm{visc2}$$

Ensures:
- conservation of momentum
- conservation of angular momentum
- strictly dissipative term

UV_VIS4 Same logic applied twice $\qquad (B_M \leftrightarrow \mathrm{visc4})$

$$-\boldsymbol{\nabla}_h \cdot \langle \mathbf{u}'_h \mathbf{u}'_h \rangle = -\frac{1}{\mathrm{Hz}} \boldsymbol{\nabla}_h \cdot (B_M \ \mathrm{Hz} \ \boldsymbol{\sigma}') \,, \qquad \boldsymbol{\sigma}' = \boldsymbol{\sigma}(\boldsymbol{\nabla}_h \cdot \boldsymbol{\sigma}(\mathbf{u}_h))$$

Closures of Smagorinsky type

UV_VIS_SMAGO, UV_VIS2

Turbulent viscosity coefficient

$$A_M = C_M\,(\Delta x \Delta y)\,\sqrt{(\partial_x u)^2 + (\partial_y v)^2 + 2(\partial_y u + \partial_x v)^2}$$

By default $C_M = \frac{1}{10}$ (parameter horcon in routif hvisc_coef)

TS_DIF_SMAGO, UV_VIS2

Turbulent diffusion coefficient

$$A_S = C_S\,(\Delta x \Delta y)\,\sqrt{(\partial_x u)^2 + (\partial_y v)^2 + 2(\partial_y u + \partial_x v)^2}$$

By default $C_S = \frac{1}{12}$ (parameter horcon in routif hdiff_coef)

Diffusion tournée (TS_MIX_ISO, TS_MIX_GEO)

TS_DIF2

Under small slopes assumption $\quad \left(\text{i.e. } \frac{\|\boldsymbol{\nabla}_h \rho\|}{\partial_z \rho} \ll 1\right) \quad$ we have

$$-\boldsymbol{\nabla}_h \cdot \langle \mathbf{u}_h' X' \rangle = \boldsymbol{\nabla} \cdot (\mathbf{R} \boldsymbol{\nabla} X), \qquad \mathbf{R} = \begin{pmatrix} A_x & 0 & A_x \alpha_x \\ 0 & A_y & A_y \alpha_y \\ A_x \alpha_x & A_y \alpha_y & A_x \alpha_x^2 + A_y \alpha_y^2 \end{pmatrix}$$

$$\alpha_m = -\left(\frac{\partial_m \rho}{\partial_z \rho}\right), \; A_x \leftrightarrow \text{diff3u}, \; A_y \leftrightarrow \text{diff3v}$$

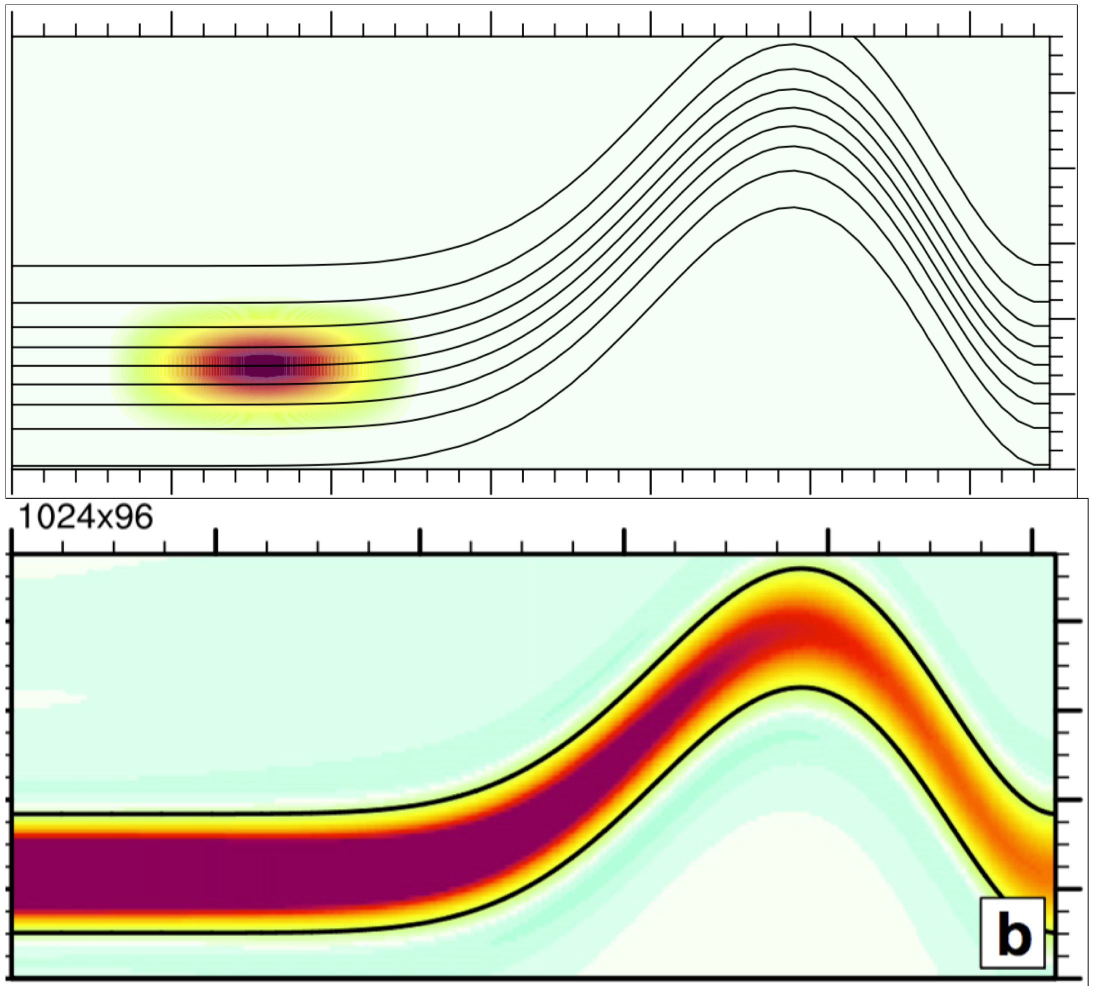
TS_DIF4

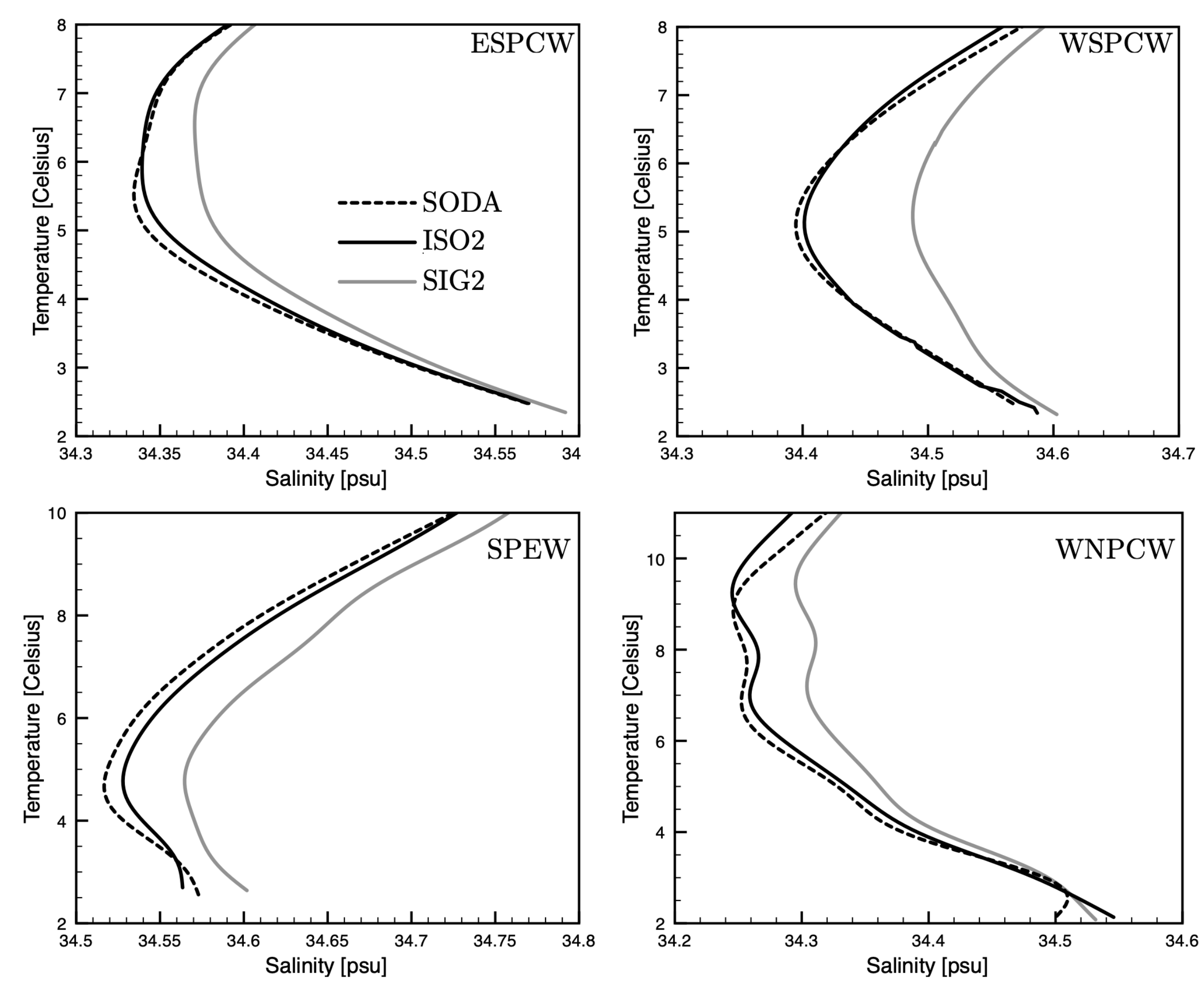
$$-\boldsymbol{\nabla}_h \cdot \langle \mathbf{u}_h' X' \rangle = -\boldsymbol{\nabla} \cdot (\mathbf{R}' \boldsymbol{\nabla} (\, \boldsymbol{\nabla} \cdot (\mathbf{R}' \boldsymbol{\nabla} X) \,)), \quad \mathbf{R}' = \begin{pmatrix} \sqrt{B_x} & 0 & \sqrt{B_x} \alpha_x \\ 0 & \sqrt{B_y} & \sqrt{B_y} \alpha_y \\ \sqrt{B_x} \alpha_x & \sqrt{B_y} \alpha_y & \sqrt{B_x} \alpha_x^2 + \sqrt{B_y} \alpha_y^2 \end{pmatrix}$$

TS_MIX_IMP : Method of stabilizing correction

$$\left\{ \begin{array}{rcl} X^\star & = & X^n + \Delta t \, \mathcal{D}(X^n) \\ X^{n+1} & = & X^\star + \Delta t \, \partial_z \left\{ \text{Akz}(z) \left(\partial_z X^{n+1} - \partial_z X^n \right) \right\} \end{array} \right.$$

Rotated diffusion : illustration and limitations

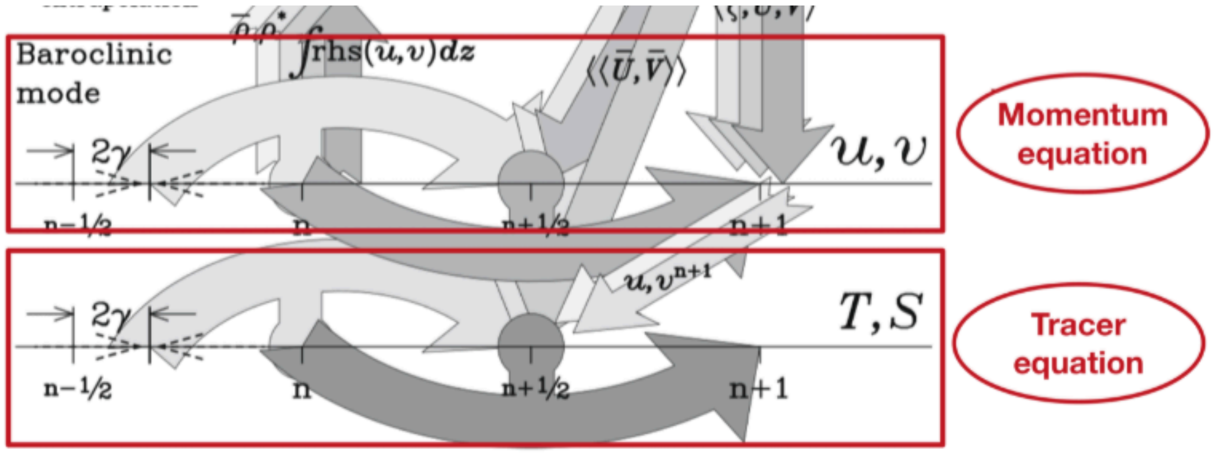




Limitations :

- Based on the small slope hypothesis
- Non-monotonous diffusion (cross-term)
- Assumes a low frequency evolution of isopycnes

In practice



on a step of Euler

hmix_coef.F : coefficients

step3d_t.F

t3dmix.F:
- *t3dmix_ISO.F (isop or geo)*
- *t3dmix_S.F*

t3dmix_tridiagonal.h

step3d_uv2.F

uv3dmix.F:
- *uv3dmix_S.F*
- *uv3dmix_GP.F*
- *uv3dmix4_S.F*
- *uv3dmix4_GP.F*