CROCO – Training Barcelonnette 2023

Model equations

CROCO
Coastal and Regional Ocean COmmunity model

CROCO equations

CROCO solves several approximations of Navier-Stokes equations in planar or spherical geometry

- ‘classic’ Hydrostatic Boussinesq mode,

CROCO equations

CROCO
Coastal and Regional Ocean COmmunity model

- Hydrostatic Boussinesq mode:

advection | Coriolis | pressure gradient | forcing terms | diffusive terms

- momentum balance

$$\frac{\partial u}{\partial t} + \vec{\nabla}.(\vec{\mathbf{v}}u) - fv = -\frac{\partial \phi}{\partial x} + \mathcal{F}_u + \mathcal{D}_u$$

$$\frac{\partial v}{\partial t} + \vec{\nabla}.(\vec{\mathbf{v}}v) + fu = -\frac{\partial \phi}{\partial y} + \mathcal{F}_v + \mathcal{D}_v$$

- evolution of tracers (e.g. T, S)

$$\frac{\partial C}{\partial t} + \vec{\nabla}.(\vec{\mathbf{v}}C) = \mathcal{F}_C + \mathcal{D}_C$$

- hydrostatic balance

$$\frac{\partial \phi}{\partial z} = -\frac{\rho g}{\rho_0}$$

- continuity

$$\vec{\nabla}.\vec{\mathbf{v}} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

- equation of state

$$\rho = \rho(T, S, P)$$

- boundary conditions

$$\frac{\mathrm{d}\zeta(\mathbf{x_H}, t)}{\mathrm{dt}} = w(z=\zeta) \qquad p(\mathbf{x_H}, z=\zeta, t) = p_{atm} \qquad \mathbf{v}(z=-H) = \mathbf{0}$$

=> prognostic variables : *u, v, T, S, ζ*
diagnostic variables : *w, p or Φ, ρ*

CROCO equations

CROCO
Coastal and Regional Ocean COmmunity model

CROCO solves several approximations of Navier-Stokes equations in planar or spherical geometry

- ‘classic’ Hydrostatic Boussinesq mode,

- Quasi-hydrostatic (QH) mode: alleviating the traditional approximation that neglects the Coriolis components proportional to the cosine of latitude,

=> adding a non-hydrostatic pressure component that is solved diagnostically

- Non-Boussinesq (NBQ) mode: pseudo-acoustic mode that allows computation of the non-hydrostatic pressure within a non-Boussinesq approach (Auclair et al., 2018),

=> additional prognostic equation of vertical velocity, replacing the hydrostatic balance

- Wave averaged-current coupled equations (vortex force formalism),

=> additional wave-induced terms

CROCO equations

Coastal and Regional Ocean COmmunity model

- Quasi-hydrostatic (QH) Boussinesq mode

  => alleviating the traditional approximation that neglects the Coriolis components proportional to the cosine of latitude
  => These hyp. become weak near the equator or in motions with a strong vertical component (e.g., convection)

$$\frac{\partial u}{\partial t} + \vec{\nabla}.(\vec{\mathbf{v}}u) - fv + \tilde{f}w = -\frac{\partial \phi}{\partial x} + \mathcal{F}_u + \mathcal{D}_u$$

$$\frac{\partial v}{\partial t} + \vec{\nabla}.(\vec{\mathbf{v}}v) + fu = -\frac{\partial \phi}{\partial y} + \mathcal{F}_v + \mathcal{D}_v$$

$$\frac{\partial \phi}{\partial z} = -\frac{\rho g}{\rho_0} + \tilde{f}u$$

$\tilde{f}(x,y)$: Non-traditional Coriolis parameter $2\Omega cos\phi$

=> in practice in the code: the non-traditional term is introduced as a correction to density (in the density computation subroutine rho_eos)

CROCO equations

Coastal and Regional Ocean COmmunity model

- Non-hydrostatic non-Boussinesq (NBQ) mode

  => acoustic waves are solved explicitly

$$\frac{\partial \rho u}{\partial t} + \vec{\nabla}.(\rho \vec{\mathbf{v}} u) - \rho f v - \rho \tilde{f} w = -\frac{\partial P}{\partial x} + \lambda \frac{\partial \vec{\nabla}.\vec{\mathbf{v}}}{\partial x} + \mathcal{F}_u + \mathcal{D}_u$$

$$\frac{\partial \rho v}{\partial t} + \vec{\nabla}.(\rho \vec{\mathbf{v}} v) + \rho f u = -\frac{\partial P}{\partial y} + \lambda \frac{\partial \vec{\nabla}.\vec{\mathbf{v}}}{\partial y} + \mathcal{F}_v + \mathcal{D}_v$$

$$\frac{\partial \rho w}{\partial t} + \vec{\nabla}.(\rho \vec{\mathbf{v}} w) - \rho \tilde{f} u = -\frac{\partial P}{\partial z} - \rho g + \lambda \frac{\partial (\vec{\nabla}.\vec{\mathbf{v}})}{\partial z} + \mathcal{F}_w + \mathcal{D}_w$$

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla}.(\rho \vec{\mathbf{v}})$$

$$\frac{\partial \xi}{\partial t} = w_f|_{z=\xi} - \vec{\mathbf{v}}|_{z=\xi}.\vec{\nabla}\xi$$

$$\frac{\partial \rho C}{\partial t} = -\vec{\nabla}.(\rho \vec{\mathbf{v}} C) + \mathcal{F}_C + \mathcal{D}_C$$

λ is the second (bulk) viscosity, associated with compressibility (it can be used to damp acoustic waves)

$$\rho = \rho_s(T,S,P) + \overbrace{\left.\frac{\partial \rho}{\partial P}\right|_{T,S} \delta P}^{\rho_f = c_s^{-2} P_f} + O(\delta P^2)$$

$$P = \underbrace{P_{atm} + \int_z^{\xi} (\rho_s - \rho_0) g \, dz'}_{SLOW} + \underbrace{\rho_0 g(\xi - z) + \overbrace{\delta P}^{P_f}}_{FAST}$$

c_s is the speed of sound

$\delta P = P_f$ is the nonhydrostatic pressure.

CROCO equations

- Wave averaged-current coupled equations (vortex force formalism)

$$\xi^c = \xi + \hat{\xi}$$ ξ^c is a composite sea level,

$$\phi^c = \phi + \hat{\phi}$$ ϕ^c absorbs the Bernoulli head $\hat{\phi}$,

$$\vec{\mathbf{v}}_L = \vec{\mathbf{v}} + \vec{\mathbf{v}}_S$$ $\vec{\mathbf{v}}_L$ is the wave-averaged Lagrangian velocity, Stokes drift $\vec{\mathbf{v}}_S$

$\mathcal{F}^W{}_u, \mathcal{F}^W{}_v, \mathcal{F}^W{}_C$: wave forcing terms (bottom streaming, breaking acceleration)

$\mathcal{D}_u, \mathcal{D}_v, \mathcal{D}_C$: diffusive terms (including wave-enhaced bottom drag and mixing)

$\mathcal{F}_u, \mathcal{F}_v, \mathcal{F}_C$: forcing terms

$$\frac{\partial u}{\partial t} + \vec{\nabla}.\left(\vec{\mathbf{v}}_L u\right) - f v_L = -\frac{\partial \phi^c}{\partial x} + \left(u_S \frac{\partial u}{\partial x} + v_S \frac{\partial v}{\partial x}\right) + \mathcal{F}_u + \mathcal{D}_u + \mathcal{F}^W{}_u$$

$$\frac{\partial v}{\partial t} + \vec{\nabla}.\left(\vec{\mathbf{v}}_L v\right) + f u_L = -\frac{\partial \phi^c}{\partial y} + \left(u_S \frac{\partial u}{\partial y} + v_S \frac{\partial v}{\partial y}\right) + \mathcal{F}_v + \mathcal{D}_v + \mathcal{F}^W{}_v$$

$$\frac{\partial \phi^c}{\partial z} + \frac{\rho g}{\rho_0} = \vec{\mathbf{v}}_S . \frac{\partial \vec{\mathbf{v}}}{\partial z}$$

$$\frac{\partial C}{\partial t} + \vec{\nabla}.\left(\vec{\mathbf{v}}_L C\right) = \mathcal{F}_C + \mathcal{D}_C + \mathcal{F}^W{}_C$$

$$\vec{\nabla}.\vec{\mathbf{v}}_L = 0$$

$$\rho = \rho(T, S, P)$$

CROCO equations

Coastal and Regional Ocean COmmunity model

CROCO solves Reynolds-averaged equations :

$$\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\vec{\nabla}\vec{u} + f\vec{k}\times\vec{u} + \frac{\rho}{\rho_0}g\vec{k} = -\frac{\vec{\nabla}P}{\rho_0} + \nu\vec{\nabla}^2\vec{u}$$

Time variation | Advection (inertia) | Rotation | Gravity | Pressure gradient | Forcings + Dissipation

Reynolds averaging $u = \overline{u} + u'$

'large', resolved scales

'small', unresolved scales

$$\frac{\partial \overline{u_i}}{\partial t} + \overline{u_j\frac{\partial u_i}{\partial x_j}} + f\mathbf{k}\times\overline{u_i} + \frac{\overline{\rho}}{\rho_0}g\mathbf{k} = -\frac{1}{\rho_0}\frac{\partial \overline{P}}{\partial x_j} + \nu\frac{\partial^2\overline{u_i}}{\partial x_i\partial x_j}$$

$$\frac{\partial \overline{u_i}}{\partial t} + \overline{u_j}\frac{\partial \overline{u_i}}{\partial x_j} + f\mathbf{k}\times\overline{u_i} + \frac{\overline{\rho}}{\rho_0}g\mathbf{k} = -\frac{1}{\rho_0}\frac{\partial \overline{P}}{\partial x_j} + \nu\frac{\partial^2\overline{u_i}}{\partial x_i\partial x_j} - \frac{\partial \overline{u_i'u_j'}}{\partial x_j}$$

Advection for the averaged flow

Reynolds stress
= effect of subgrid-scale turbulence

→ Turbulence closure

$$\begin{aligned}
\mathcal{F}_u &= \frac{\partial}{\partial z}\left(K_{Mv}\frac{\partial u}{\partial z}\right) + \nabla_h(K_{Mh}.\nabla_h u)\\
\mathcal{F}_v &= \frac{\partial}{\partial z}\left(K_{Mv}\frac{\partial v}{\partial z}\right) + \nabla_h(K_{Mh}.\nabla_h v)\\
\mathcal{S}_T &= \frac{\partial}{\partial z}\left(K_{Tv}\frac{\partial T}{\partial z}\right) + \nabla_h(K_{Th}.\nabla_h T)\\
\mathcal{S}_S &= \frac{\partial}{\partial z}\left(K_{Sv}\frac{\partial S}{\partial z}\right) + \nabla_h(K_{Sh}.\nabla_h S)
\end{aligned}$$

Vertical mixing | Horizontal diffusion

CROCO equations

Turbulence closure

$$
\begin{aligned}
\mathcal{F}_u &= \frac{\partial}{\partial z}\left(K_{Mv}\frac{\partial u}{\partial z}\right) + \nabla_h(K_{Mh}.\nabla_h u) \\
\mathcal{F}_v &= \frac{\partial}{\partial z}\left(K_{Mv}\frac{\partial v}{\partial z}\right) + \nabla_h(K_{Mh}.\nabla_h v) \\
\mathcal{S}_T &= \frac{\partial}{\partial z}\left(K_{Tv}\frac{\partial T}{\partial z}\right) + \nabla_h(K_{Th}.\nabla_h T) \\
\mathcal{S}_S &= \frac{\partial}{\partial z}\left(K_{Sv}\frac{\partial S}{\partial z}\right) + \nabla_h(K_{Sh}.\nabla_h S)
\end{aligned}
$$

Vertical mixing　　　Horizontal diffusion

Vertical Mixing: K_{Mv}, K_{Tv}, K_{Sv}

- Local Turbulence closure: GLS, k-kl (MY2.5), k-ε, κ-ω, etc. *[e.g. Warner et al, 2005, Ocean Modelling]*
- Non local K-profile parameterization (KPP) *[Large et al, 1994, Rev. of Geophysics]*

Horizontal diffusion: K_{Mh}, K_{Th}, K_{Sh}

- Explicit diffusion
- Implicit (comes from the advective scheme)

CROCO equations

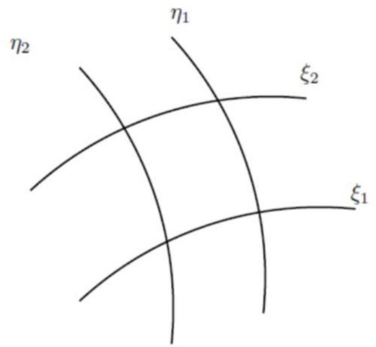
Coastal and Regional Ocean COmmunity model

- CROCO equations are expressed in terms of

  - orthogonal curvilinear horizontal coordinates (ξ, η)

$$(ds)_\xi = \left(\frac{1}{m}\right) d\xi$$

$$(ds)_\eta = \left(\frac{1}{n}\right) d\eta$$



  - the terrain- and free surface-following vertical coordinate $-1 \leqslant s \leqslant 0$

$$z^{(0)}(x,y,s) = h \cdot \frac{h_c \cdot s + h \cdot C(s)}{h + h_c},$$

$$z = z^{(0)} + \zeta(1 + z^{(0)}/h)$$

$$H_z \equiv \frac{\partial z}{\partial s}$$

$$C(s) \equiv C[S(s)],$$

$$C(S) = \frac{\exp(\theta_b S) - 1}{1 - \exp(-\theta_b)}, \quad S(s) = \frac{1 - \cosh(\theta_s s)}{\cosh(\theta_s) - 1}$$

CROCO equations

- For the hydrostatic Boussinesq mode, CROCO equations are written

$$\frac{\mathcal{D}u}{\mathcal{D}t} - \widehat{\mathcal{F}}\,v = -\frac{H_z}{n}\cdot\frac{1}{\rho_0}\cdot\left.\frac{\partial p}{\partial \xi}\right|_z + \mathcal{G}_u$$

$$\frac{\mathcal{D}v}{\mathcal{D}t} + \widehat{\mathcal{F}}u = -\frac{H_z}{m}\cdot\frac{1}{\rho_0}\cdot\left.\frac{\partial p}{\partial \eta}\right|_z + \mathcal{G}_v$$

$$\frac{\mathcal{D}q}{\mathcal{D}t} = \mathcal{G}_q, \quad q \in \{\text{const}, \Theta, S, \ldots\}$$

$$\rho = \rho_{\text{ EOS}}(\Theta, S, P) \quad \text{with } P = \rho_0 g(\zeta - z).$$

$$\frac{\mathcal{D}*}{\mathcal{D}t} = \frac{\partial}{\partial t}\left(\frac{H_z}{mn}*\right) + \frac{\partial}{\partial \xi}\left(\frac{H_z u}{n}*\right) + \frac{\partial}{\partial \eta}\left(\frac{H_z v}{m}*\right) + \frac{\partial}{\partial s}\left(\frac{\omega_s}{mn}*\right)$$

CROCO equations

CROCO
Coastal and Regional Ocean COmmunity model

- For the hydrostatic Boussinesq mode, CROCO equations are written

$$\frac{\mathcal{D}u}{\mathcal{D}t} - \widehat{\mathcal{F}}v = -\frac{H_z}{n}\cdot\frac{1}{\rho_0}\cdot\left.\frac{\partial p}{\partial \xi}\right|_z + \mathcal{G}_u$$

$$\frac{\mathcal{D}v}{\mathcal{D}t} + \widehat{\mathcal{F}}u = -\frac{H_z}{m}\cdot\frac{1}{\rho_0}\cdot\left.\frac{\partial p}{\partial \eta}\right|_z + \mathcal{G}_v$$

$$\widehat{\mathcal{F}} = \frac{H_z}{mn}\left\{f + mn\cdot\left(v\frac{\partial(1/n)}{\partial\xi} - u\frac{\partial(1/m)}{\partial\eta}\right)\right\}$$

$$\frac{\mathcal{D}q}{\mathcal{D}t} = \mathcal{G}_q, \quad q \in \{\text{const}, \Theta, S, \ldots\}$$

$$\rho = \rho_{\text{ EOS}}(\Theta, S, P) \quad \text{with } P = \rho_0 g(\zeta - z).$$

$$\frac{\mathcal{D}*}{\mathcal{D}t} = \frac{\partial}{\partial t}\left(\frac{H_z}{mn}*\right) + \frac{\partial}{\partial\xi}\left(\frac{H_z u}{n}*\right) + \frac{\partial}{\partial\eta}\left(\frac{H_z v}{m}*\right) + \frac{\partial}{\partial s}\left(\frac{\omega_s}{mn}*\right)$$

CROCO equations

- For the hydrostatic Boussinesq mode, CROCO equations are written

$$\frac{\mathcal{D}u}{\mathcal{D}t} - \widehat{\mathcal{F}} v = -\frac{H_z}{n} \cdot \frac{1}{\rho_0} \cdot \left.\frac{\partial p}{\partial \xi}\right|_z + \mathcal{G}_u$$

$$-\left.\frac{\partial p}{\partial \xi}\right|_z = -g\rho|_{s=0} \cdot \frac{\partial \zeta}{\partial \xi} - g \int_s^0 \left[\frac{\partial z}{\partial s} \cdot \left.\frac{\partial \rho}{\partial \xi}\right|_s - \frac{\partial \rho}{\partial s} \cdot \left.\frac{\partial z}{\partial \xi}\right|_s \right] ds'$$

$$\frac{\mathcal{D}v}{\mathcal{D}t} + \widehat{\mathcal{F}} u = -\frac{H_z}{m} \cdot \frac{1}{\rho_0} \cdot \left.\frac{\partial p}{\partial \eta}\right|_z + \mathcal{G}_v$$

$$\frac{\mathcal{D}q}{\mathcal{D}t} = \mathcal{G}_q, \quad q \in \{\text{const}, \Theta, S, \ldots\}$$

$$\rho = \rho_{\text{ EOS}}(\Theta, S, P) \quad \text{with } P = \rho_0 g(\zeta - z).$$

$$\frac{\mathcal{D}*}{\mathcal{D}t} = \frac{\partial}{\partial t}\left(\frac{H_z}{mn}*\right) + \frac{\partial}{\partial \xi}\left(\frac{H_z u}{n}*\right) + \frac{\partial}{\partial \eta}\left(\frac{H_z v}{m}*\right) + \frac{\partial}{\partial s}\left(\frac{\omega_s}{mn}*\right)$$

CROCO equations : summary

- CROCO can solve several approximations of Navier-Stokes equations in planar or spherical geometry :
  - for most applications, the classic hydrostatic mode is relevant
  - for applications involving equatorial dynamics, small-scales, coupling with waves, other modes may be necessary

- CROCO solves Reynolds-averaged form of equations :
  - effects of resolved scales are explicitly represented
  - effects of unresolved scales are parameterized : horizontal/vertical mixing schemes has to be chosen

- CROCO equations are formulated with boundary-fitted coordinates :
  - (ξ, η) in the horizontal could be coastline-following (rarely used as such in practice)
  - s in the vertical which is free-surface- and terrain-following

Numerics

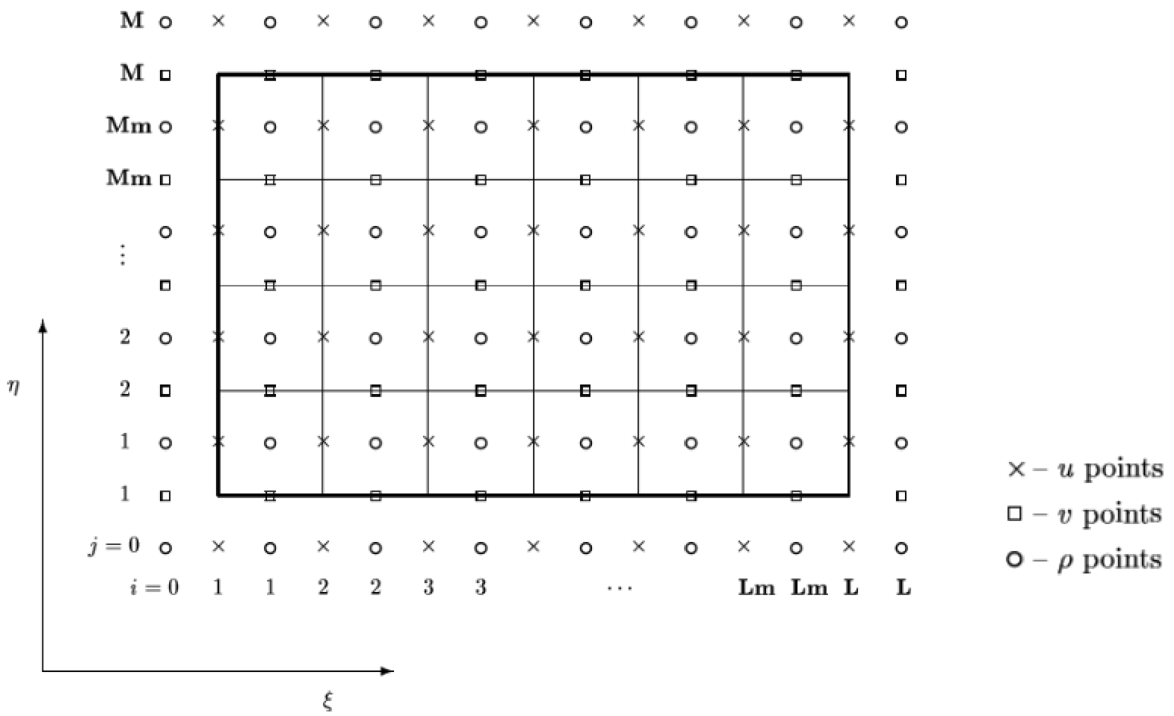
CROCO
Coastal and Regional Ocean COmmunity model

- Grid
- Numerical schemes
- Physical parameterizations
- Time stepping

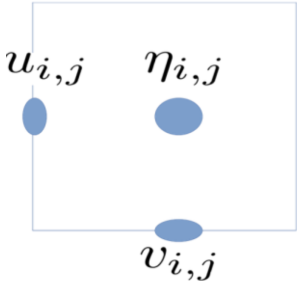
Grid

Arakawa-C structured grid

staggered difference is 4 times more accurate than non-staggered and improves the dispersion relation because of reduced use of averaging operators



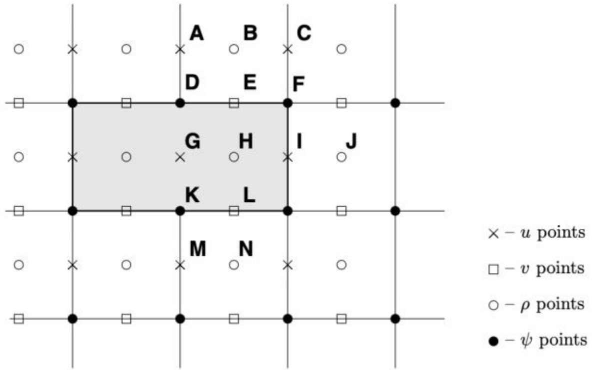
CROCO indexing



Grid

- land masking (MASKING)

Variables within the masked region are set to zero by multiplying by the mask for either the u, v or rho points :

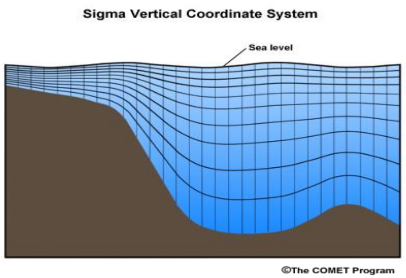


- wetting-Drying scheme (WET_DRY) : cancel the outgoing momentum flux (not the incoming) from a grid cell if its total depth is below a threshold value (critical depth Dcrit between 5 and 20 cm according to local slope; Dcrit min and max adjustable in param.h)

Grid

Coastal and Regional Ocean COmmunity model

vertical S-coordinates



$$z^{(0)}(x,y,s) = h \cdot \frac{h_c \cdot s + h \cdot C(s)}{h + h_c},$$

$$z = z^{(0)} + \zeta(1 + z^{(0)}/h)$$

=> Stretching possible in the surface and bottom boundary layers:
Cs=f(**θ**s, **θ**b*)* stretching function, h_C : thickness controlling stretching
The effects of **θ**s, **θ**b*, hc, and N* can be tested using the Matlab script croco_tools/Preprocessing_tools/test_vgrid.m

=> take care of pressure gradient errors on steep slopes: tuncation errors are made from calculating the baroclinic pressure gradients across sharp topographic changes such as the continental slope

=> solution: smoothing the topography using a nonlinear filter and a criterium: $r = \Delta h / h < 0.2$
(+ high-order numerical schemes)

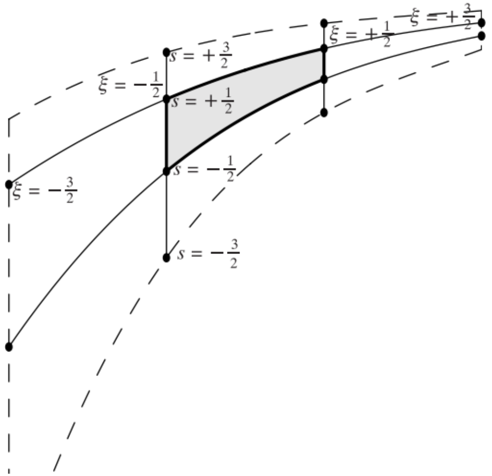
Numerical schemes: Pressure gradient

- In s coordinates, the horiz. pressure gradient consists of two large terms that tend to cancel

$$-\left.\frac{\partial p}{\partial \xi}\right|_z = -g\rho|_{s=0}\cdot\frac{\partial\zeta}{\partial\xi} - g\int_s^0\left[\frac{\partial z}{\partial s}\cdot\left.\frac{\partial\rho}{\partial\xi}\right|_s - \frac{\partial\rho}{\partial s}\cdot\left.\frac{\partial z}{\partial\xi}\right|_s\right]ds'$$

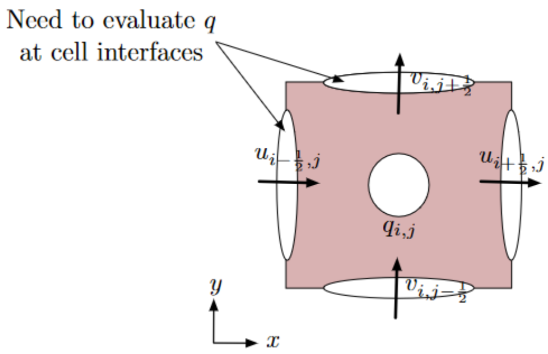
- Interference of the discretization errors of these terms induces pressure gradient errors and drives spurious currents

- CROCO uses the pressure gradient algorithm by Shchepetkin, McWilliams JGR2003

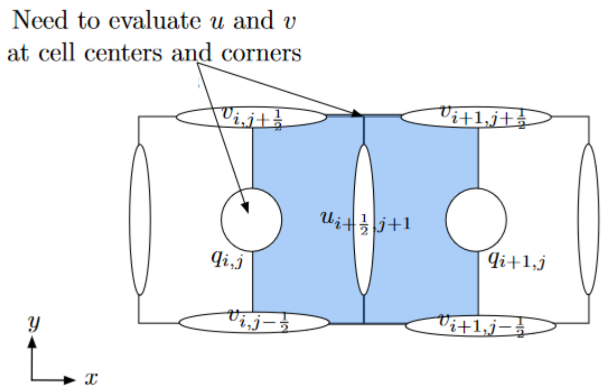


Advection discretisation = interpolation problem

Tracers

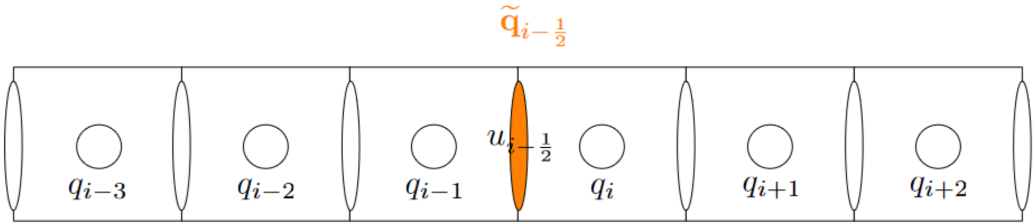


Momentum



Numerical schemes: Advection

Horizontal advection [HADV_C2, HADV_UP3, HADV_C4, HADV_UP5, HADV_C6]



$$\partial_x (uq)|_{x=x_i} = \frac{1}{\Delta x_i} \left\{ u_{i+1/2}\widetilde{q}_{i+1/2} - u_{i-1/2}\widetilde{q}_{i-1/2} \right\}$$

$$\widetilde{q}^{C2}_{i-1/2} = \frac{q_i + q_{i-1}}{2}$$

$$\widetilde{q}^{C4}_{i-1/2} = (7/6)\widetilde{q}^{C2}_{i-1/2} - (1/12)(q_{i+1} + q_{i-2})$$

$$\widetilde{q}^{UP3}_{i-1/2} = \widetilde{q}^{C4}_{i-1/2} + \mathrm{sign}(1/12, u_{i-1/2})(q_{i+1} - 3q_i + 3q_{i-1} - q_{i-2})$$

$$\widetilde{q}^{C6}_{i-1/2} = (8/5)\widetilde{q}^{C4}_{i-1/2} - (19/60)\widetilde{q}^{C2}_{i-1/2} + (1/60)(q_{i+2} + q_{i-3})$$

$$\widetilde{q}^{UP5}_{i-1/2} = \widetilde{q}^{C6}_{i-1/2} - \mathrm{sign}(1/60, u_{i-1/2})(q_{i+2} - 5q_{i+1} + 10q_i - 10q_{i-1} + 5q_{i-2} - q_{i-3})$$

Numerical schemes: Advection

Horizontal advection : properties of linear schemes

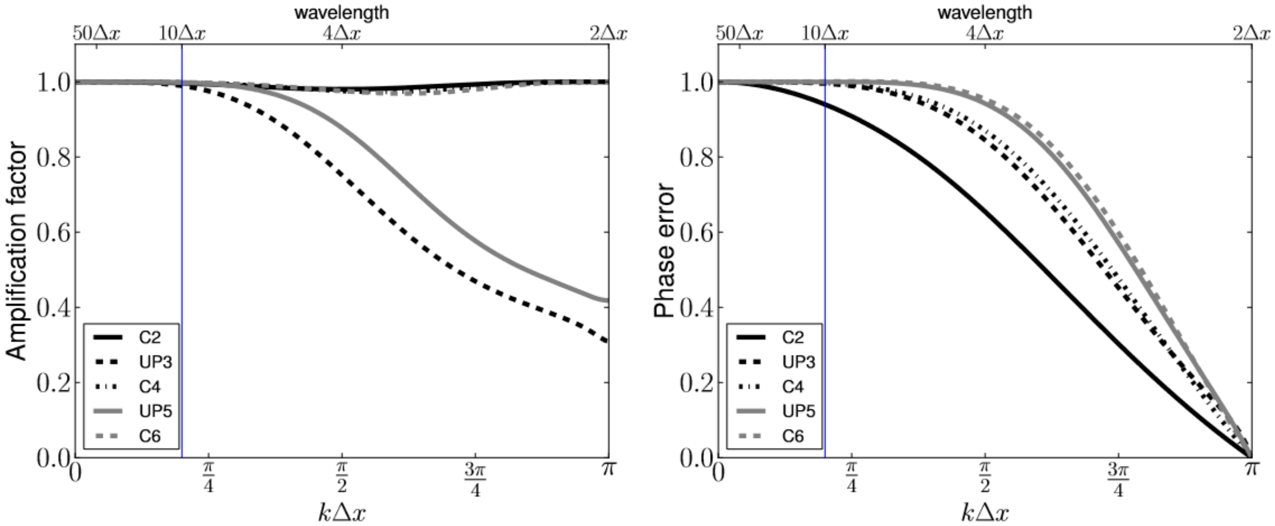
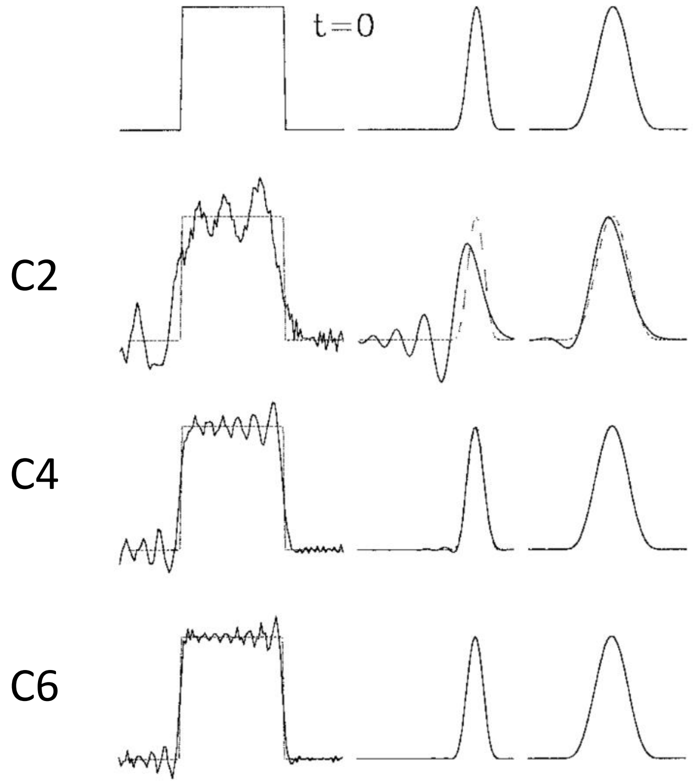


Figure: Amplification error (left) and phase error (right) for linear advection schemes of order 2 to 6.

Numerical schemes: Advection



- Spurious over/under shoots appear where the advected signal varies sharply
- Explicit diffusion is needed with even-order schemes
- Tend to promote odd-order schemes (UP3 and UP5)
- Monotonicity can be enforced with TVD corrections or by using WENO5 schemes

Numerical schemes: Advection

Coastal and Regional Ocean COmmunity model

- Available schemes in CROCO

Advices for choosing:

- Not choosing => take default
- Look at sources of errors
- Consider the physics (scales, need of positivity…)
- Consider computation time

Equation	horizontal	vertical
Moment 3D (UV_ADV) (+ eq W_ en NBQ)	UV_HADV_TVD UV_HADV_C2 **UV_HADV_UP3** UV_HADV_C4 UV_HADV_UP5 UV_HADV_C6 UV_HADV_WENO5	UV_VADV_TVD UV_VADV_C2 **UV_VADV_SPLINES** UV_VADV_WENO5
Traceurs	**TS_HADV_UP3** TS_HADV_RSUP3 TS_HADV_C4 TS_HADV_WENO5 TS_HADV_UP5 TS_HADV_C6 TS_HADV_RSUP5	TS_VADV_TVD TS_VADV_C2 TS_VADV_SPLINES **TS_VADV_AKIMA** TS_VADV_WENO5
Moment 2D	**M2_HADV_UP3** M2_HADV_C2	

Physical parameterizations

- Vertical mixing

turbulence => parameterized as mixing

$$\left\langle X'w' \right\rangle = -K_X(z)\partial_z \left\langle X \right\rangle, \qquad X = u, v, T, S$$

depends on:

- Surface forcing (wind stress, buoyancy)

$$\begin{aligned} -\langle \mathbf{u}_h' w' \rangle_{\text{sfc}} &= \boldsymbol{\tau}/\rho_o \\ -\langle T' w' \rangle_{\text{sfc}} &= Q_H/(\rho_o C_{p,o}) \end{aligned}$$

- Bottom forcing (bottom stress, buoyancy)

$$\begin{aligned} -\langle \mathbf{u}_h' w' \rangle_{\text{bot}} &= \boldsymbol{\tau}_b/\rho_o \\ -\langle T' w' \rangle_{\text{bot}} &= 0 \end{aligned}$$

- Interior conditions (current shear instability, stratification)

$$\begin{aligned} -\langle \mathbf{u}_h' w' \rangle (z) &= K_m(z)\partial_z \langle \mathbf{u}_h \rangle \\ -\langle T' w' \rangle (z) &= K_s(z)\partial_z \langle T \rangle \end{aligned}$$

Physical parameterizations

Coastal and Regional Ocean COmmunity model

- Surface forcing (wind stress, buoyancy)

$$
\begin{aligned}
-\langle \mathbf{u}_h' w' \rangle_{\text{sfc}} &= \boldsymbol{\tau}/\rho_o \\
-\langle T' w' \rangle_{\text{sfc}} &= Q_H/(\rho_o C_{p,o})
\end{aligned}
$$

=> directly read forcing files with heat, freshwater, and momentum fluxes
=> or use bulk parameterizations

BULK_FLUX	Activate bulk formulation for surface turbulent fluxes (by default, COARE3p0 parametrizarion is used)
BULK_ECUMEV0	Use ECUMEv0 bulk formulation instead of COARE3p0 formulation
BULK_ECUMEV6	Use ECUMEv6 bulk formulation instead of COARE3p0 formulation
BULK_WASP	Use WASP bulk formulation instead of COARE3p0 formulation
BULK_GUSTINESS	Add in gustiness effect on surface wind module. Can be used for both bulk parametrizations.
BULK_LW	Add in long-wave radiation feedback from model SST

=> Additional functionalities

SFLUX_CFB	Activate current feedback on ... (Renault et al., 2020)
CFB_STRESS	... surface stress (used by default when SFLUX_CFB is defined)
CFB_WIND_TRA	... surface tracers (used by default when SFLUX_CFB is defined)
SST_SKIN	Activate skin sst computation (Zeng & Beljaars, 2005)

QCORRECTION	Activate heat flux correction around model SST (if BULK_FLUX is undefined)
SFLX_CORR	Activate freshwater flux correction around model SSS (if BULK_FLUX is undefined)
ANA_DIURNAL_SW	Activate analytical diurnal modulation of short wave radiations (only appropriate if there is no diurnal cycle in data)

Physical parameterizations

CROCO
Coastal and Regional Ocean COmmunity model

- Bottom forcing (bottom stress, buoyancy)

$$
\begin{aligned}
-\langle \mathbf{u}'_h w' \rangle_{\mathrm{bot}} &= \boldsymbol{\tau}_b/\rho_o \\
-\langle T' w' \rangle_{\mathrm{bot}} &= 0
\end{aligned}
$$

=> specified in **croco.in**

```
bottom_drag:     RDRG [m/s],  RDRG2,  Zob [m],  Cdb_min, Cdb_max
                 3.0d-04      0.d-3   0.d-3     1.d-4    1.d-1
```

Available formulations:

- Quadratic friction with log-layer (Zob !=0)
- Quadratic friction with Cd = cst (RDRG2 > 0)
- Linear friction (RDRG)

=> Additional functionalities

LIMIT_BSTRESS	Bottom stress limitation forstability
BSTRESS_FAST	Bottom stress computed in step3d_fast
BBL	Bottom boundary layer parametrization

Physical parameterizations

- Interior conditions (current shear instability, stratification)

$$
\begin{aligned}
-\langle \mathbf{u}_h' w' \rangle (z) &= K_m(z)\partial_z \langle \mathbf{u}_h \rangle \\
-\langle T' w' \rangle (z) &= K_s(z)\partial_z \langle T \rangle
\end{aligned}
$$

Available options :

ANA_VMIX	Analytical definition
BVF_MIXING	Brunt-Vaisaleafrequency based
LMD_MIXING	K-profile parametrisation
GLS_MIXING	Generic lengthscale parametrisation

Physical parameterizations

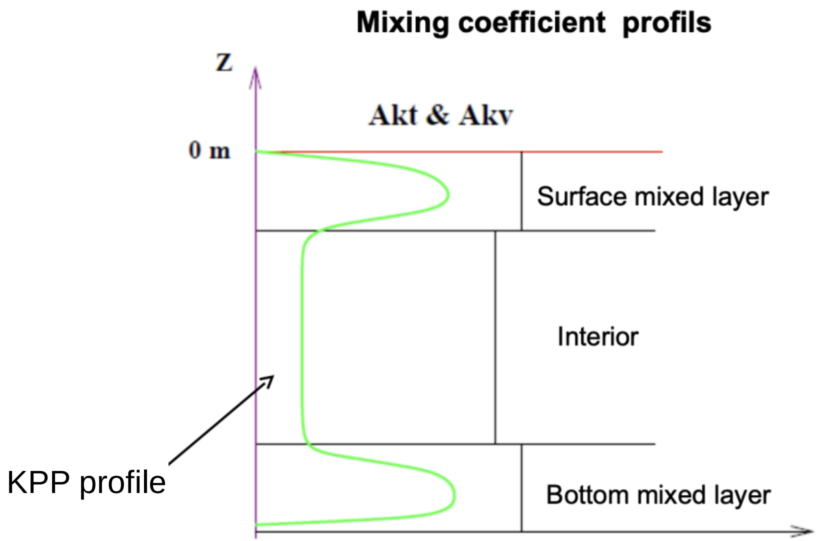
CROCO
Coastal and Regional Ocean COmmunity model

- Interior conditions (current shear instability, stratification)

$$-\langle \mathbf{u}_h' w' \rangle (z) = K_m(z) \partial_z \langle \mathbf{u}_h \rangle$$
$$-\langle T' w' \rangle (z) = K_s(z) \partial_z \langle T \rangle$$

KPP-related options :

LMD_MIXING	K-profile parametrisation
LMD_SKPP	Activate surface boundary layer KPP mixing
LMD_SKPP2005	Activate surface boundary layer KPP mixing (2005 version)
LMD_BKPP	Activate bottom boundary layer KPP mixing
LMD_BKPP2005	Activate bottom boundary layer KPP mixing (2005 version)
LMD_RIMIX	Activate shear instability interior mixing
LMD_CONVEC	Activate convection interior mixing
LMD_DDMIX	Activate double diffusion interior mixing
LMD_NONLOCAL	Activate nonlocal transport for SKPP
LMD_LANGMUIR	Activate Langmuir turbulence mixing



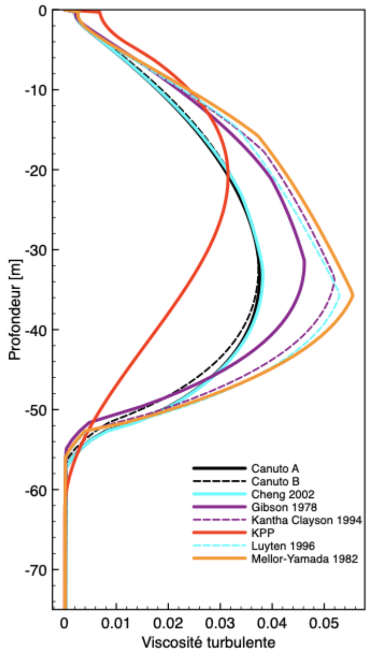
Physical parameterizations

- Interior conditions (current shear instability, stratification)

$$-\langle \mathbf{u}'_h w' \rangle (z) = K_m(z)\partial_z \langle \mathbf{u}_h \rangle$$
$$-\langle T' w' \rangle (z) = K_s(z)\partial_z \langle T \rangle$$

GLS schemes : several possibilities

GLS_MIXING	Activate Generic Length Scale scheme, default is k-epsilon (see below)
GLS_KOMEGA	Activate K-OMEGA (OMEGA=frequency of TKE dissipation) originating from Kolmogorov (1942)
GLS_KEPSILON	Activate K-EPSILON (EPSILON=TKE dissipation) as in Jones and Launder (1972)
GLS_GEN	Activate generic model of Umlauf and Burchard (2003)
CANUTO_A	Option for CANUTO A stability function (default, see below)
GibLau_78	Option for Gibson & Launder, 1978 stability function
MelYam_82	Option for Mellor & Yamada, 1982 stability function
KanCla_94	Option for Kantha & Clayson, 1994 stability function
Luyten_96	Option for Luyten, 1996 stability function
CANUTO_B	Option for CANUTO B stability function
Cheng_02	Option for Cheng, 2002 stability function



Physical parameterizations

- Interior conditions (current shear instability, stratification)

$$-\langle \mathbf{u}'_h w' \rangle (z) = K_m(z)\partial_z \langle \mathbf{u}_h \rangle$$
$$-\langle T' w' \rangle (z) = K_s(z)\partial_z \langle T \rangle$$

Available options :

ANA_VMIX	Analytical definition
BVF_MIXING	Brunt-Vaisaleafrequency based
LMD_MIXING	K-profile parametrisation
GLS_MIXING	Generic lengthscale parametrisation

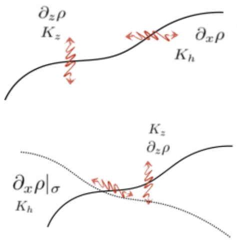
=> Advices for choosing:

- KPP assumes that turbulence in the boundary layer is in equilibrium with surface and bottom fluxes => true for large scale models
- TKE models explicitly treat temporal high frequency in the BL, role of horizontal terms in TKE equation almost not studied…
- For coastal applications, the scheme should : respond to local forcing, respond rapidly to surface and bottom fluxes => GLS-type scheme preferred

Physical parameterizations: Horizontal mixing

CROCO
Coastal and Regional Ocean COmmunity model

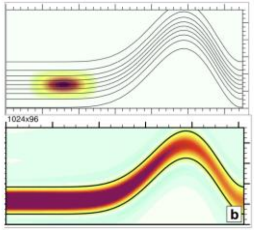
- Horizontal mixing



Note: a fraction of dissipation of energy and mixing arises through the horizontal advection operator (even more with non-rotated advection schemes)

=> Explicit lateral momentum mixing may be only useful when implicit dissipation in UV_HADV_UP3 is not large enough to account for subgrid-scale turbulence resulting from large shear currents (for example in the case of western boundary currents). In this case, Smagorinsky parametrization is recommended (define UV_VIS2)

=> The options are preselected in set_global_definitions.h for compliance with Advection options.



Momentum $-\nabla_h \cdot \langle \mathbf{u}'_h \mathbf{u}'_h \rangle$

UV_MIX_GEO	Activate mixing on geopotential (constant depth) surfaces
UV_MIX_S	Activate mixing on iso-sigma (constant sigma) surfaces
UV_VIS2	Activate Laplacian horizontal mixing of momentum
UV_VIS4	Activate Bilaplacian horizontal mixing of momentum
UV_VIS_SMAGO	Activate Smagorinsky parametrization of turbulent viscosity (only with UV_VIS2)
UV_VIS_SMAGO3D	Activate 3D Smagorinsky parametrization of turbulent viscosity

Tracers $-\nabla_h \cdot \langle \mathbf{u}'_h X' \rangle$

TS_MIX_ISO	Activate mixing along isopycnal (isoneutral) surfaces
TS_MIX_GEO	Activate mixing along geopotential surfaces
TS_MIX_S	Activate mixing along iso-sigma surfaces
TS_DIF2	Activate Laplacian horizontal mixing of tracer
TS_DIF4	Activate Bilaplacian horizontal mixing of tracer
TS_MIX_IMP	Activate stabilizing correction of rotated diffusion (used with TS_MIX_ISO and TS_MIX_GEO)

CROCO time stepping

- CROCO is a free-surface model :
  - fast scales due to the strong restoring effect at the free surface,
  - slow scales due to the weak restoring effect in the ocean interior.

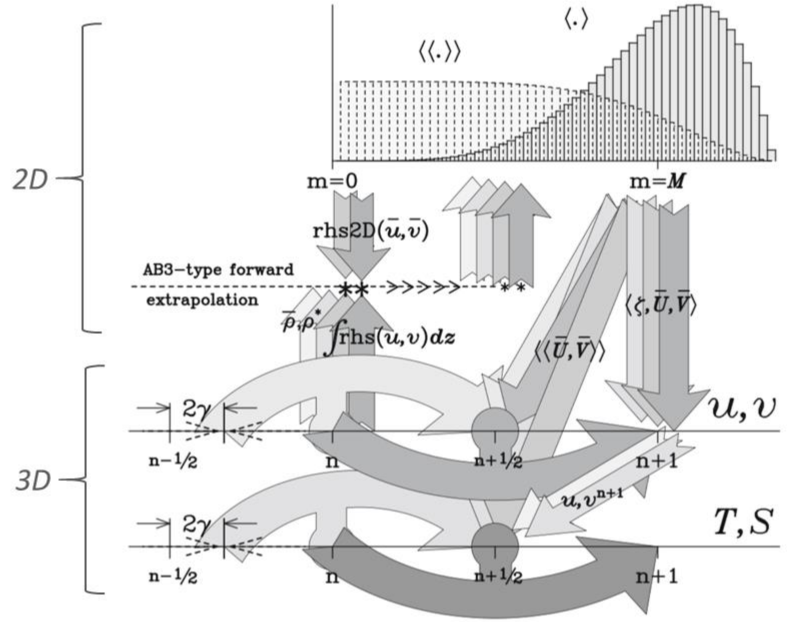
=> CROCO uses a time-splitting strategy :

- a set of equations representing fast scales are splitted from the full equations,
- the fast scales equations are integrated in time
  => using an efficient time-stepping scheme
  => a time step sub-multiple of the main one $\delta t = \Delta t / N$

- for the hydrostatic Boussinesq mode, the fast scales variables are 2D 'barotropic',

- for NBQ mode, the barotropic mode solver is replaced by a fully 3D fast mode solver, resolving all waves down to acoustic waves (with sound speed that can be decreased to the maximum wave speed one wants to solve).

CROCO time stepping



the barotropic mode (with $D = \zeta + H$ and $D\bar{\mathbf{u}} = \int_{-H}^{\zeta} \mathbf{u}_h \, dz$)

$$\begin{cases} \partial_t \zeta &= -\nabla_h \cdot D\bar{\mathbf{u}} \\ \partial_t (D\bar{u}) &= Df\bar{v} - \nabla_h \cdot (D\bar{u}\bar{\mathbf{u}}) - gH\partial_x\zeta - (g/2)\partial_x\zeta^2 + D\mathcal{F}_{3D}^{(u)} \\ \partial_t (D\bar{v}) &= -Df\bar{u} - \nabla_h \cdot (D\bar{v}\bar{\mathbf{u}}) - gH\partial_y\zeta - (g/2)\partial_y\zeta^2 + D\mathcal{F}_{3D}^{(v)} \end{cases}$$

CROCO numerics: summary

High-order numerics

High-order numerical schemes: pressure gradient, 3rd and 5th-order advection schemes

Rotated tensors to reduce diapycnal mixing

KPP and GLS mixing parameterizations

Split-explicit time-stepping